UNCLASSIFIED J.W. Gordon

TD-9

LA-2459
Copy No.

C.3

AEC RESEARCH AND DEVELOPMENT REPORT

LOS ALAMOS SCIENTIFIC LABORATORY
OF THE UNIVERSITY OF CALIFORNIA ○ LOS ALAMOS   NEW MEXICO

SINGLE STAGE NUCLEAR ROCKET STUDY
(Title Unclassified)

UNCLASSIFIED

LEGAL NOTICE

UNCLASSIFIED

This document consists of __77__ pages

No._____of _138_ copies, Series __A__

LOS ALAMOS SCIENTIFIC LABORATORY
OF THE UNIVERSITY OF CALIFORNIA   LOS ALAMOS   NEW MEXICO

REPORT WRITTEN: March 1960

REPORT DISTRIBUTED: July 17, 1961

PUBLICLY RELEASABLE
Per_____ . FSS-16 Date: 9-19-95
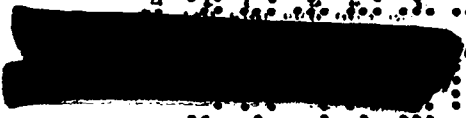By_____ . CIC-14 Date: 10-17-95

SINGLE STAGE NUCLEAR ROCKET STUDY

(Title Unclassified)

by

E. Haddad
W. Anderson

Classification changed to UNCLASSIFIED
by authority of the U. S. Atomic Energy Commission
Per Jacob B. Rahn, Chief Be FC W. ___, 1-15-76
By REPORT LIBRARY Marcia Adams 1-83-70

UNCLASSIFIED

UNCLASSIFIED
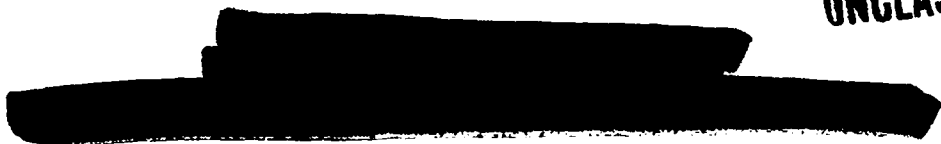
UNCLASSIFIED

UNCLASSIFIED

UNCLASSIFIED

ABSTRACT

This report describes a study which was carried out primarily on ground-launched, single-stage, hydrogen-propelled nuclear rockets. The missions considered were flights into circular orbits and ballistic flights.

Particular emphasis was given to determine the effect of missile trajectory, for a specified mission, on payload. The types of trajectories considered were the powered-all-the-way (PAW) ascent, the ballistic ascent, and the elliptic ascent. In the PAW missions, the missile was flown from the ground, along a trajectory that maximized payload, into a prescribed circular orbit. The PAW results were then compared, for a particular mission, with ballistic and elliptic ascent calculations. The results show that most of the missions studied were very sensitive to the type of trajectory flown, thus clearly demonstrating the gains to be made by using a nuclear rocket motor which has a recycle capability.

UNCLASSIFIED

Mission performance results have been obtained for various single-stage nuclear systems, but with emphasis on missions which require large payloads in orbit. Present day technology indicates that a nuclear system having a launch weight of ~400,000 lb would be capable of placing a payload in excess of 40,000 lb into a 300-mi circular orbit.

UNCLASSIFIED

CONTENTS

UNCLASSIFIED

UNCLASSIFIED

Page

TABLES

ILLUSTRATIONS

UNCLASSIFIED

ILLUSTRATIONS (Continued)

I. Introduction

The purpose of this report is to describe a study which was carried out primarily on ground-launch, single-stage, hydrogen-propelled nuclear rockets. The performance of a nuclear rocket containing a fuel made up of hydrogen and methane (3 mole %) as well as a brief discussion of a vehicle employing tank staging are also included in this report. The missions considered in this study were flights into circular orbits and ballistic flights. Sections I and II are a review and extension of the work presented by K. W. Ford.[1] The emphasis in Ref. 1 was to fly a given pay-load a maximum distance. The emphasis here is to fly a rocket from the ground, along a trajectory that maximizes payload, into a prescribed circular orbit. The types of trajectories considered were the powered-all-the-way (PAW) ascent, the ballistic ascent, and the elliptic ascent. In the PAW ascent the vehicle engine is operated continuously until the desired orbit is achieved. In the ballistic ascent the vehicle is carried to a summit (by means of a propulsion and free-flight phase) which lies near the desired orbit. At the summit, the vehicle must, in a second propulsion period, produce the velocity increment required for transfer into the desired orbit. In the well-known Hohmann[2] transfer ellipse (elliptic ascent), the vehicle

first attains circular velocity at a relatively low

altitude (taken to be 100 statute miles in this report).

Subsequently, the vehicle is accelerated to the perigee

velocity of the transfer ellipse, at whose apogee an

additional short burst of power is required for entering

the satellite orbit.

In this work, the philosophy has been to consider in

detail the aspects of a particular mission problem only

if it is justified by present day knowledge. For this

reason, an accurate formulation of the equations for air

drag, air pressure, trajectory, and trajectory integration,

etc., are used, whereas the equations that pertain to

rocket component weights have been simplified as far as

possible. The question of the type and weight of the

rocket motor, when the reactor weight is not available

from a detailed engineering study, has been avoided in a

given mission problem by lumping together the reactor and

payload weight. As more data become available, the

estimates of tank, structure, and other weights given here

will change, thereby necessitating changes in the results

given in this report.

The following few paragraphs will describe the

formulation used to solve a given mission problem in a

qualitative way. Details of the formulation are given

in sections II to VI. Conclusions are given in section VII.

Section II, Staging of the Rocket on the Ground, is concerned with the determination of component masses and sizes and propellant flow rate. In section II the takeoff weight is written as the sum of 10 component weights:

1. W_L, payload plus nose cone plus guidance, given

2. W_F^{H2}, hydrogen propellant weight

3. W_F^{CH4}, methane propellant weight

4. W_T^{H2}, hydrogen tank weight, calculated from hydrogen propellant weight

5. W_T^{CH4}, methane tank weight, calculated from methane propellant weight

6. W_S^{H2}, hydrogen structure weight, given as a fixed fraction of W_T^{H2}

7. W_S^{CH4}, methane structure weight, given as a fixed fraction of W_T^{CH4}

8. W_P^T, weight of turbopumps and plumbing, written as a function of pump discharge pressure and volume flow rate

9. W_R, weight of reactor plus pressure shell plus gimbals, given

10. W_N, weight of the nozzle, written as a function of nozzle throat area.

The exhaust velocity can be calculated from the chamber temperature and pressure. From this, the mass flow rate and total mass can be calculated which will give the specified initial acceleration.

Section III is concerned with the trajectory calculation through the atmosphere. The trajectory is vertical for a time t_k (kickover time), at which time the thrust angle to the vertical is changed to β_k (kickover angle). From the time t_k until the rocket passes through the sensible atmosphere (height h_2), the rocket flies along a gravity-turn trajectory (thrust vector parallel to velocity). The effects of air pressure and air density are included in this calculation and are obtained from an exponential approximation. Mach number and drag coefficient are calculated as functions of altitude from the equations in section III B of this report.[3] The effect of air pressure on thrust and the variation of the gravitational force with altitude are taken into account.

In section IV the rocket is powered all the way (PAW) from the height h_2, along a trajectory that maximizes

payload, into a prescribed circular orbit. For this part of the trajectory the magnitude of the thrust as well as the mass flow rate were kept constant. The mathematical formulation used in section IV follows closely the work of L. Turner on minimum time of flight trajectories.[4]

In section V the rocket is flown from the height h_2, along a trajectory that maximizes payload, into a 100-mi circular orbit. The rocket is flown from the 100-mi orbit to any other specified circular orbit by means of a Hohmann transfer ellipse. Section VI gives a brief discussion of the ballistic ascent trajectory, and conclusions are given in section VII.

II. Staging of the Rocket on the Ground

A. Weights of Components

1. Payload weight, W_L, given (includes nose cone and guidance) in lb

2. Propellant weight (hydrogen), W_F^{H2}, given in lb

3. Propellant weight (methane), W_F^{CH4}, given in lb

4. Hydrogen tank weight,

$$W_T^{H2} = \frac{Q_1 (K + 2)}{(1 - a)(4/3 + K)} \left(\frac{\rho_T}{\rho_{H_2}}\right) (1 - F) W_F^T$$

5. Methane tank weight,

$$W_T^{CH4} = \frac{Q_2(K + 2)}{(1 - a)\ (4/3 + K)}\left(\frac{\rho_T}{\rho_{CH_4}}\right)(F)\ W_F^T$$

The equations for the tank weights were derived on the assumption of a particular tank configuration. The configuration chosen was a tank of radius R_T, with the cylindrical straight section of length $\ell_T = KR_T$, and hemispherical endcaps. The over-all length of the rocket is $L = KR_T + 2R_T$. The tank walls are made of steel of constant thickness. In the foregoing equations,

ρ_T = steel density in lb/ft^3 (value used was 494)

ρ_{H_2} = hydrogen density in lb/ft^3 (value used was 4.43)

ρ_{CH_4} = methane density in lb/ft^3 (value used was 43.68)

F = weight percent of methane

W_F^T = total fuel weight (hydrogen plus methane)

a = fraction of tank volume left for air space (ullage factor)

Q_1 = $2(f_s P_T^{H2})/\sigma$

Q_2 = $2(f_s P_T^{CH4})/\sigma$

where

f_s = factor of safety

$$p_T^{H2} = \text{hydrogen tank pressure in psi (gas overpressure plus hydrostatic head)}$$

$$p_T^{CH4} = \text{methane tank pressure in psi (gas overpressure plus hydrostatic head)}$$

$$\sigma = \text{yield strength in psi}$$

In this work K = 10, a = 0.03, f_s = 1.39 and was taken from LA-1870,[5] and σ = 150,000. The value used for the overpressure in the tanks was 25 psi (based on the Atlas design), the value of the hydrogen hydrostatic head was assumed to be 10 psi, and the value of the methane hydrostatic head was assumed to be 35 psi.

6. Hydrogen structure weight assumed is

$$W_S^{H2} = 0.15 \ W_T^{H2} \ \text{in lb}$$

7. Methane structure weight assumed is
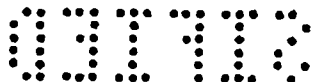
$$W_S^{CH4} = 0.15 \ W_T^{CH4} \ \text{in lb}$$

8. The total pump weight (W_P^T) is

$$W_P^T = W_P^{H2} + W_P^{CH4} \ \text{in lb}$$

Since the present state of hydrogen pump development is in its infancy and no methane pump development is in progress, no attempt was made to estimate separate pump weights. What was done was to assume that

$$W_P^T \simeq W_P^{H2} = \frac{C_p(1-F)\dot{W}_F^T (P_d)^{1/2}}{\rho_{H_2}} \qquad (1)$$

since only problems containing small admixtures of methane were considered in this report. In Eq. 1

$\dot{W}_F^T$ = total propellant flow rate in lb/sec

ρ_{H_2} = hydrogen propellant density

P_d = pump discharge pressure in lb/in.^{2}

and

C_p = 1.28 (see page 287 of Ref. 3)

9. Reactor weight plus pressure shell plus reflector. No attempt was made to calculate reactor weights when detailed engineering data were unavailable. When this was the case, the calculations were performed by lumping together the reactor and payload weights.

10. The nozzle weight is

$$W_N = C_N A_t$$

where

A_t = throat area in in.^{2}

C_N = 6.8 (the value used for the nozzle coefficient is thought to be conservative)[6]

B.   Mass Flow Rate, Thrust, Takeoff Weight*

1.   Exit pressure. For a gamma law gas and adiabatic expansion, the ratio of exit area to throat area is related to the ratio of exit pressure to chamber pressure by (see Ref. 7, page 61)

$$\frac{A_e}{A_t} = \left(\frac{2}{k+1}\right)^{(k-1)^{-1}}\left(\frac{P_e}{P_c}\right)^{-k^{-1}}\left\{\left(\frac{k+1}{k-1}\right)\left[1 - \left(\frac{P_e}{P_c}\right)^{(k-1)/k}\right]\right\}^{-1/2} \quad (2)$$

where k is the ratio of the specific heat at constant pressure to the specific heat at constant volume. In this problem the area ratio is specified, as is the chamber pressure P_c. Hence this equation yields a value for the exit pressure P_e.

2.   Exhaust velocity. The exhaust velocity for a perfect, infinitely expanding nozzle is

$$v_e^0 = \left[\left(\frac{2k}{k-1}\right)\left(\frac{g_0 \overline{R} T_c}{M_{av}}\right)\right]^{-1/2} \quad (3)$$

where

   $\overline{R}$ = the universal gas constant

   $\overline{R}$ = 2782 ft-lb/lb-mole °K

   T_c = the chamber temperature in °K

---

*Portions of section B were taken from an internal memorandum by K. W. Ford, a LASL consultant.

g_0 = a constant = 32.17 ft/sec^2

M_{av} = the average molecular weight, defined by

$$M_{av} = \frac{\sum\limits_{i} \bar{n}_i M_i}{\sum\limits_{i} \bar{n}_i} \qquad (4)$$

where

$\bar{n}_i$ = mole percent of constituent i

M_i = molecular weight of constituent i

The actual exhaust velocity is

$$V_e = V_e^0 C_{NL} \left[1 - \left(\frac{P_e}{P_c} \right)^{(k-1)/k} \right]^{1/2} \qquad (5)$$

where C_{NL} is a nozzle efficiency factor, set equal to 0.97 for the problems discussed in this report.

3. <u>Effective exhaust velocity and specific impulse.</u>
To the exhaust velocity must be added a term due to finite exit pressure and finite air pressure, which we call the "pressure velocity,"

$$V_p = \left(1 - \frac{P_a}{P_e} \right) C* \left(\frac{A_e}{A_t} \right) \left(\frac{P_e}{P_c} \right) \qquad (6)$$

where P_a is air pressure and $C*$, the characteristic velocity, is given by

$$C* = \frac{\left[(kg_0 \bar{R}T_c)/M_{av}\right]^{1/2}}{k\left[2/(k+1)\right]^{(k+1)/2(k-1)}} \tag{7}$$

The specific impulse is

$$I_{sp} = \frac{(V_e + V_p)}{g_0} \tag{8}$$

where $V_e + V_p$ is the effective exhaust velocity. Because of the flow separation in the nozzle, the ratio P_a/P_e cannot become arbitrarily large. In this problem we limit the ratio to 3. If P_e is calculated to be less than $P_a/3$, we arbitrarily replace $(1 - P_a/P_e)$ in Eq. 6 by -2.

4. **Mass flow rate.** The effective exhaust velocity and given desired initial takeoff acceleration, $g_0 a_0$, specify the mass flow rate,

$$\dot{W}_F^T = \frac{g_0 W_0 (1 + a_0)}{(1 - \bar{\beta})(V_e + V_p)} = \frac{W_0 (1 + a_0)}{(1 - \bar{\beta})(I_{sp})} \tag{9}$$

where $\bar{\beta}$ is the fraction of the propellant used to drive the pump. This fraction is assumed to be exhausted at negligible velocity.

5. **Throat area.** The throat area is given by

$$A_t = \frac{C*\dot{W}_F^T}{(P_c g_0)} \tag{10}$$

6. <u>Reactor power</u>. The reactor power in Mev is given by

$$P_R = C_R \dot{W}_F^T \, \Delta T \tag{11}$$

where ΔT = the temperature drop of the gas through the reactor, in °C

C_R = 7.21 x 10^{-3} megajoules/lb °C

The alteration of specific heat due to admixed CH_4 and the question of dissociation and recombination are not taken into account.

7. <u>Launch weight equation</u>. An explicit formula for total takeoff mass W_0 may be arrived at from the formulas given above, and is

$$W_0 = \cfrac{W_L + W_R + W_F^T \left\{ \cfrac{(K+2)Q_1 \rho_T}{(1-a)(4/3+K)} \left[\cfrac{1.15 \left(\left[Q_2/Q_1 \right] \rho_{H_2} - \rho_{CH_4} \right)^{F} + 1.15 \rho_{CH_4}}{\rho_{H_2} \rho_{CH_4}} \right] + 1 \right\}}{1 - \cfrac{g_0(1+a_0)}{(1-\bar{\beta})(V_e + V_p^0)} \left[\cfrac{(1-F)C_p P_d^{1/2}}{\rho_{H_2}} + \cfrac{C_N C^*}{(P_c g_0)} \right]} \tag{12}$$

Once W_0 is determined from Eq. (12), all component weights can be found as well as the mass flow rate $\dot{W}_F$. The total burning time, if all the fuel is used, is

$$t_B = W_F^T \frac{(1 - \alpha)}{\dot{W}_F^T} \tag{13}$$

where α is the fraction of unused propellant at burnout (holdover).

With a slight modification of Eq. 12, the equations given in this section can be used to determine the performance of a chemical system. To do this, Eq. 1 must be changed so that W_p^T is the sum of two pump weights, which in turn introduces an additional term in Eq. 12.

III. Trajectory Calculation through the Atmosphere for a Non-Rotating Circular Earth

For this part of the flight the rocket is flown vertically for a time t_k (kickover time), at which time the thrust angle to the vertical is changed to β_k (kickover angle). From the time t_k until the rocket passes through the sensible atmosphere, the rocket flies along a gravity-turn trajectory (thrust vector parallel to velocity). The equations of motion for this part of the flight (see Fig. 1) are

$$\dot{V} = \frac{(T - D)g_0}{W} - g\cos\beta \tag{14}$$

$$\dot{\beta} = -\left(\frac{V\sin\beta}{R} - g\frac{\sin\beta}{V}\right) \tag{15}$$

$$\dot{R} = V\cos\beta \tag{16}$$

$$\dot{\Theta} = \frac{V}{R}\sin\beta \tag{17}$$
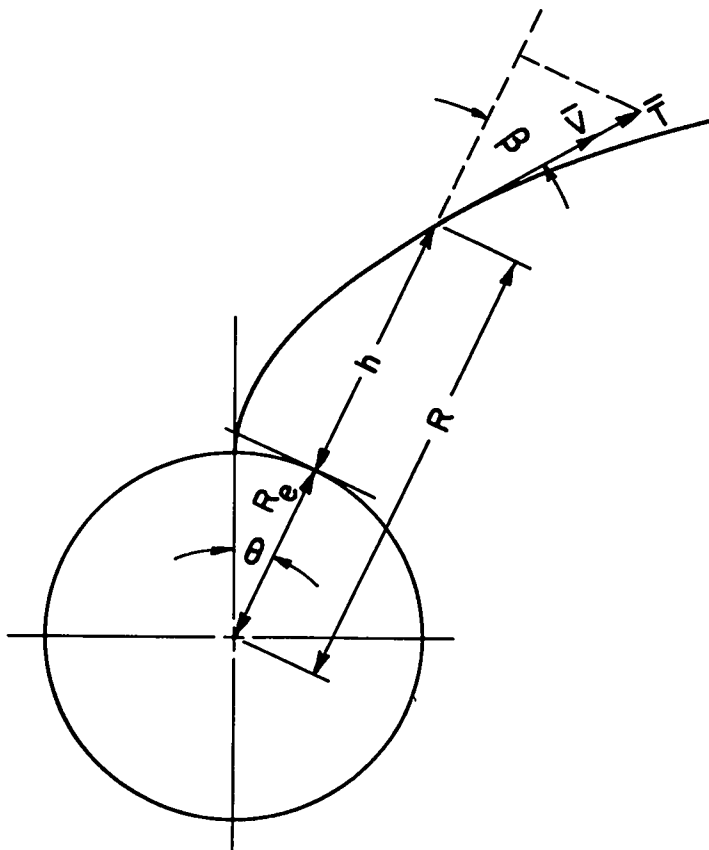
Fig. 1 Coordinate system for gravity-turn portion of
        powered flight

where

V = absolute value of the tangential velocity

β = angle the velocity vector makes with local vertical

R = distance from center of earth to missile

Θ = angle the radius vector makes with y axis

T = rocket thrust in pounds

D = drag force in pounds

In order to solve this set of simultaneous, nonlinear, first order differential equations we need to obtain Tg_0/W, Dg_0/W, and g in terms of the variables of integration. These quantities will now be evaluated.

A. Evaluation of Tg_0/W

The specific thrust (Tg_0/W) is a function of $W(t)$ and changing air pressure. The effect of $W(t)$ is to cause the specific thrust to be proportional to $\left[1-(\dot{W}_F^T/W_0)t\right]^{-1}$, and the effect of changing air pressure causes the specific thrust to be proportional to $(V_e + V_p)/(V_e + V_p^0)$. The value of V_p is

$$V_p = \frac{V_p^0\,(1 - P_a/P_e)}{(1 - 14.7/P_e)} \tag{18}$$

except that the denominator and/or the quantity in parentheses in the numerator are arbitrarily replaced by -2 if either is calculated to be less than -2. Thus Tg_0/W

can be written as

$$\frac{Tg_0}{W} = \left(\frac{Tg_0}{W}\right)_0 \left[\frac{(V_e + V_p)}{(V_e + V_p^0)}\right]\left[1 - \left(\overset{.T}{W}_F/W_0\right)t\right]^{-1} \tag{19}$$

where $(Tg_0/W)_0$ is the initial specific thrust and is

$g_0(1 + a_0)$. The value of P_a was obtained from the equation

$$P_a = 14.7\ e^{-(h/22,800)} \tag{20}$$

where h is the altitude in feet (see Fig. 1).

B. Evaluation of Dg_0/W

The equation for the specific drag (the value of D
was obtained from page 373 of Ref. 7) is

$$\frac{Dg_0}{W(t)} = \frac{\left[\pi g_0 f_D C_D D_T^2 \rho_a(h) V^2\right]}{8W(t)} \tag{21}$$

where

D_T = tank diameter in ft

ρ_a = air density in slugs/ft^3

g_0 = 32.2 ft/sec^2

V = speed in ft/sec

$W(t) = W_0 - \overset{.T}{W}_F t$

The value of ρ_a was obtained from the equation

$$\rho_a(h) = AT\ e^{-(h/22,800)} \tag{22}$$

where AT = 0.003 and h is in feet. The formulas used to obtain the drag coefficient are

$$C_D = 0.37 \qquad\qquad \text{if} \quad M < 0.9$$

$$C_D = 0.60 + 2.3(M - 1) \quad \text{if } 0.9 \leq \quad M \leq 1.1$$

$$C_D = 0.30 + (0.60/M) \qquad \text{if} \quad M > 1.1$$

where M = V/c = Mach number. Sound speed, c, is represented by

$$c = 1120 - 0.00414h \qquad \text{if} \quad h < 35,000 \text{ ft}$$

$$c = 975 \text{ ft/sec} \qquad \text{if} \quad h \geq 35,000 \text{ ft}$$

The value of c should increase again at very high altitude, but by this time, the drag is small, and an accurate value of c is not important. It is felt that the relative slope of the curve represented by these equations is correct, but the absolute values of the numbers to be assigned to the ordinate are in doubt. The coefficient f_D is introduced into the set of input parameters in order to facilitate easy changes in C_D as better information on C_D vs M becomes available. (For the problems in this report f_D was taken to be 1.)

C. Gravity Term

The gravitational acceleration is

$$g = g_0 \left(\frac{R_e}{R_e + h} \right)^2 \tag{23}$$

where R_e, the radius of the earth, is 2.0908 x 10^7 ft.

1. <u>Solution of the equations.</u> Having Tg_0/W, Dg_0/W, and g in terms of the variables of integration, values of β, V, R, and θ were obtained as a function of time by numerically solving, by means of Runge-Kutta, Eqs. 1 through 5. The calculations were carried out on an IBM 704 computer. The program for this problem is given in the appendix.

A possible flight demonstration mission to prove the possibility of nuclear rocket propulsion would be to lob a vehicle to an altitude of ~500 to ~1000 mi and then let it impact a few hundred miles from the launch pad. A reactor operating at a temperature of 2000°C and running at a power level of ~1000 Mw, Kiwi C design, has the ability to generate enough thrust for this mission. Using the equations developed in section II, the launch weight of such a vehicle was found to be ~41,000 lb. The vehicle component weight breakdown is given in Table 1. The trajectory history, obtained from the equations developed in this section, is given in Fig. 2. The input parameters used in working this problem, and which are common to all problems discussed in the report, are also given in Table 1.

TABLE 1

Mission "Lob Shot" Results

Launch weight	40,600 lb
Flow rate	79.3 lb/sec
Reactor power	1140 Mw
Reactor exit gas temp	2000°C
Pump and turbine weight	1000 lb
Nozzle weight	400 lb
Structure weight	600 lb
Tank weight	2100 lb
Reactor weight	9000 lb
Fuel weight	27,000 lb
Holdover weight	400 lb
Tank diameter	11.3 ft
Ratio of over-all length to diameter	6
Velocity at burnout	14,050 ft/sec
Maximum height	1050 mi
Total burning time	341 sec
Distance from launch pad to impact	300 mi
Payload	100 lb
Total time in air	1685 sec

Holdover	0.01	Fuel fraction used to drive pump	0.05
Ullage	0.03	Takeoff acceleration	0.35 g_0
Ratio of length to diameter	6	Effective k of hot hydrogen	1.35
Pump discharge pressure	1500 psi	Nozzle exit area to throat area	25
Chamber pressure	1070 psi	Nozzle eff factor	0.97
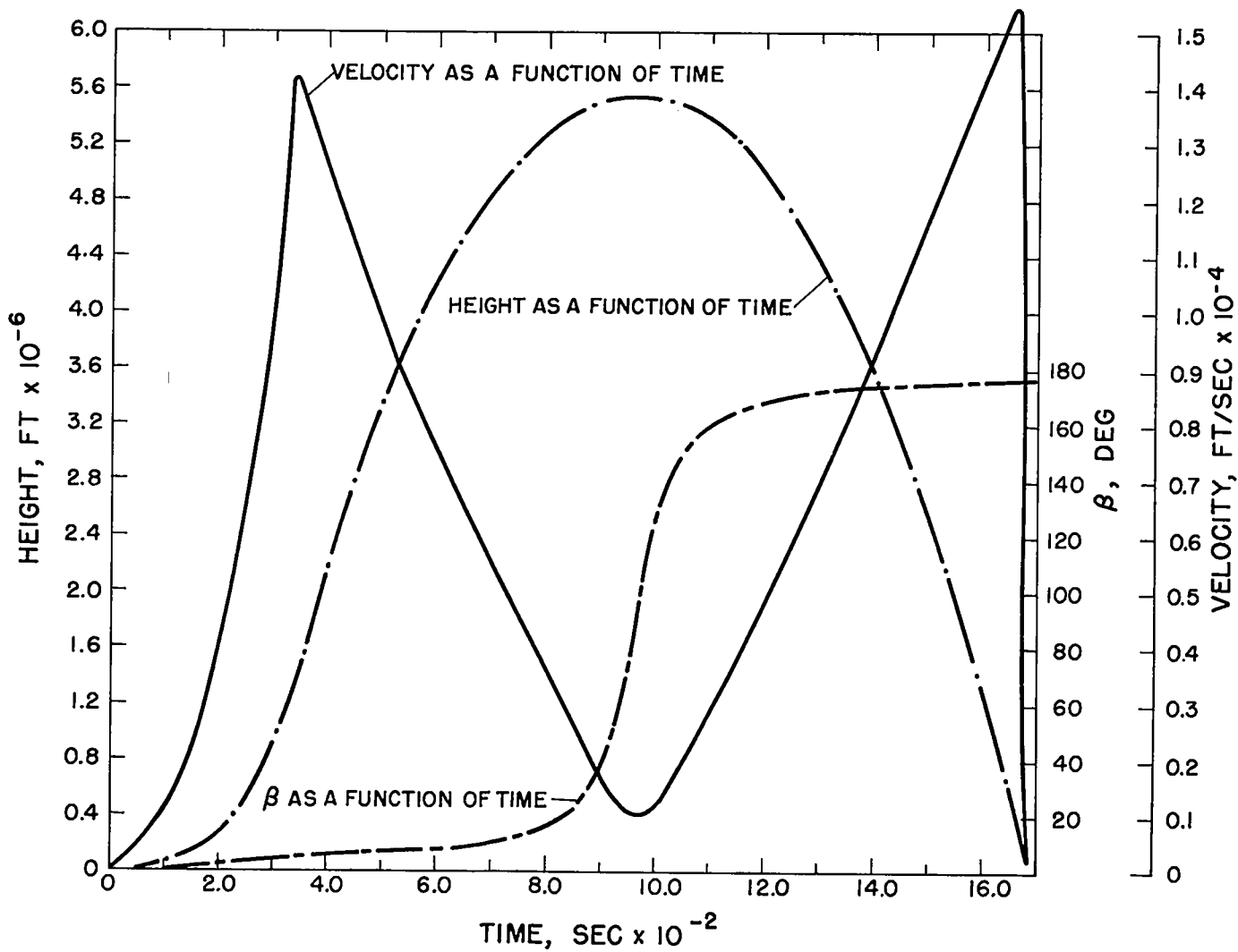
Fig. 2 Trajectory history obtained from gravity-turn equations

IV. Powered-All-the-Way Trajectory Optimization

A. Derivation of Optimization Equations

The direction of the thrust is programmed in such a way as to minimize the time required for the rocket to fly from the top of the atmosphere (h_2) to the prescribed orbit. During this portion of the flight both $\dot{W}$ and I_{sp} are kept constant. Since both $\dot{W}$ and I_{sp} are constant, minimum flight time means maximum payload. The required mathematical formulation will now be derived.

Let R and Θ be polar coordinates of a point in space with respect to an inertial coordinate system whose origin is at the center of the earth (see Fig. 3). In Fig. 3, β is the angle the velocity vector makes with the local vertical, γ the angle the thrust vector makes with the local vertical, and h the altitude of the rocket above the surface of the earth. At time t_2 a missile with mass W_{02} is at (h_2, β_2) and has velocity V_2. It is flown from h_2 and arrives at the prescribed circular orbit after a time $t' = t_3 - t_2$, which is unknown. For this portion of the flight the time scale was shifted an amount t_2 for convenience in the integrations. To obtain the actual flight time, t_2 must be added to t'. It is desired to find functions η_R and η_Θ, the specific thrusts in the direction of R and Θ (see Fig. 3), which minimize t' under

29
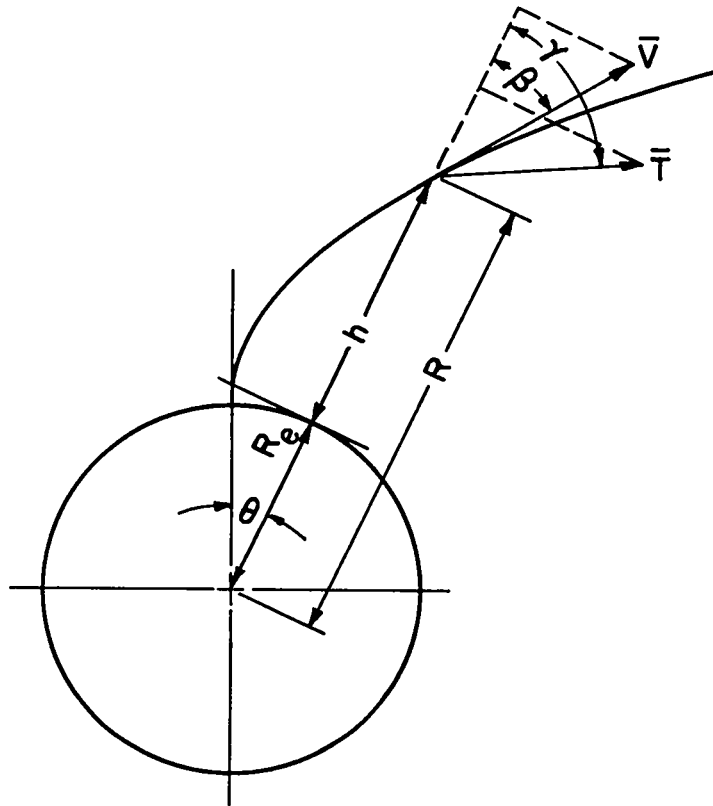
Fig. 3   Coordinate system used for trajectory optimization
         portion of flight

the given constraints. The functions η_R and η_Θ satisfy

$$\eta_R^2 + \eta_\Theta^2 = \left(\frac{\dot{W}I_{sp}g_0}{W_{02} - \dot{W}t}\right)^2 \tag{24}$$

from t = 0 to t = t', after which they are both zero. The value of t' can be determined for a specific orbit altitude from a knowledge of η_R and η_Θ.

The equations of motion are

$$\ddot{R} - R\dot{\Theta}^2 + \frac{\bar{k}}{R^2} = \eta_R \tag{25}$$

and

$$2\dot{R}\dot{\Theta} + R\ddot{\Theta} = \eta_\Theta \tag{26}$$

where

$$\bar{k} = g_0 R_e^2$$

Letting $U = \dot{R}$ and $L = R^2\dot{\Theta}$ leads to the following set of equations:

$$\frac{\dot{L}}{R} - \eta_\Theta = 0 = \psi_1 \tag{27}$$

$$\dot{U} - \frac{L^2}{R^3} + \frac{\bar{k}}{R^2} - \eta_R = 0 = \psi_2 \tag{28}$$

$$\dot{R} - U = 0 = \psi_3 \tag{29}$$

plus the equation

$$\eta_R^2 + \eta_\Theta^2 - \left(\frac{g_0\dot{W}I_{sp}}{W_0 - \dot{W}t}\right)^2 = 0 = \psi_4 \tag{30}$$

Now we wish to fly the missile from height h_2 to height h_3 in the minimum amount of time, thus expending the minimum amount of fuel. To do this we minimize the time subject to the above differential constraints and the initial conditions $R = R_2$, $U = U_2$, $L = L_2$ and the final (or end) conditions $R = R_3$, $U = U_3$, and $L = L_3$, where L_3 is the angular momentum necessary to have a circular orbit of radius R_3. The integral to be minimized is

$$I = \int_{t=0}^{t=t'} F(R, U, L, \eta_R, \eta_\Theta, \dot{U}, \dot{L}, \dot{R}, t)\, dt \qquad (31)$$

where

$$F = 1 + \sum_i \lambda_i(t)\, \Psi_i , \qquad i = 1 \text{ to } 4 \qquad (32)$$

Now in order that $\delta I = 0$, the function F must satisfy the Euler-Lagrange equations

$$\frac{d}{dt}\left(\frac{\partial F}{\partial \dot{y}_i}\right) - \frac{\partial F}{\partial y_i} = 0 \qquad (33)$$

and in addition the transversality condition[8] at t'. This condition is

$$\left[(F - \sum_i \dot{y}_i \frac{\partial F}{\partial \dot{y}_i})\, dt + \sum_i \frac{\partial F}{\partial \dot{y}_i}\, dy_i \right]_{t'} = 0 \qquad (34)$$

where dt and dy_i pertain to the final curve (circular orbit) and the coefficients of dt and dy_i pertain to the

path leading into the final curve. Substituting F into Eq.33 leads to the following set of equations:

$$\dot{\lambda}_3 = \dot{\lambda}_1 \frac{\dot{L}}{R^2} - \lambda_2 \left(\frac{3L^2}{R^4} - \frac{2\bar{k}}{R^3} \right) = 0 \tag{35}$$

$$\dot{\lambda}_1 = \lambda_1 \frac{U}{R} + 2\lambda_2 \frac{L}{R^2} = 0 \tag{36}$$

$$\dot{\lambda}_2 + \lambda_3 = 0 \tag{37}$$

$$\lambda_2 - 2\eta_R \lambda_4 = 0 \tag{38}$$

$$\lambda_1 - 2\eta_\Theta \lambda_4 = 0 \tag{39}$$

Eliminating λ_4 from Eqs. 38 and 39 and making use of Eq. 30 yields

$$\eta_R = \left[\frac{\lambda_2}{\left(\lambda_2^2 + \lambda_1^2 \right)^{1/2}} \right] \left[\frac{g_0 \dot{W} I_{sp}}{\left(W_{02} - \dot{W}t \right)} \right] \tag{40}$$

and

$$\eta_\Theta = \left[\frac{\lambda_1}{\left(\lambda_2^2 + \lambda_1^2 \right)^{1/2}} \right] \left[\frac{g_0 \dot{W} I_{sp}}{\left(W_{02} - \dot{W}t \right)} \right] \tag{41}$$

Equations 40 and 41 show that λ_2 is a direction number along the radius vector and that λ_1 is a direction number normal to the radius vector. Substituting F into Eq. 34 and writing the equation of the orbit as a function

of t, namely, R = C (a constant), L = C, and U = 0, yields

$$\left\{ \left[F - \left(\frac{\dot{L}\lambda_1}{R} \right) + \dot{U}\lambda_2 + \dot{R}\lambda_3 \right] dt \right\}_{t'} = 0 \qquad (42)$$

and since $\dot{\psi}_i = 0$, $\dot{R}_3 = 0$, Eq. 42 becomes

$$\left(\frac{\dot{L}\lambda_1}{R} + \dot{U}\lambda_2 \right)_{t'} = 1 \qquad (43)$$

Since $\dot{U} = \eta_R$ and $\dot{L} = R\eta_\theta$ at t = t', Eq. 43 can be written as

$$\left(\eta_\theta\lambda_1 + \eta_R\lambda_2 \right)_{t'} = 1 \qquad (44)$$

which is our transversality condition.

From Eqs. 40, 41, and 44 the equation for the final time (t') in terms of the directional components of the thrust is

$$t' = \left(\frac{W_{02}}{\dot{W}} \right) - g_0 I_{sp} \left(\lambda_1^2 + \lambda_2^2 \right)^{1/2} \qquad (45)$$

and since t_2 is known, the actual flight time to enter into the prescribed circular orbit is t_3, where $t_3 = t' + t_2$.

For convenience we now group the equations to be solved together. They are

$$\dot{L} - R\eta_\theta = 0$$

$$\dot{U} - \frac{L^2}{R^3} + \frac{k}{R^2} - \eta_R = 0$$

$$\dot{R} - U = 0$$

$$\dot{\lambda}_3 + \lambda_1 \frac{\eta\,\Theta}{R} - \lambda_2 \frac{3L^2}{R^4} - \frac{2\bar{k}}{R^3} = 0$$

$$\dot{\lambda}_1 - U\frac{\lambda_1}{R} + 2L\frac{\lambda_2}{R^2} = 0$$

$$\dot{\lambda}_2 + \lambda_3 = 0$$

The boundary conditions at $t = 0$ are $R = R_2$, $L = L_2$, $U = U_2$ and are given in section III (obtained from the solution of the gravity-turn equations). The boundary conditions at $t = t'$ are $R = R_3$, $U = 0$, $L = L_3 = (\bar{k}R_3)^{1/2}$. In the above set of equations we wish to solve for L, U, R, λ_1, λ_2, λ_3 as functions of time.

B. <u>Numerical Solution of the Boundary Value Problem</u>

To obtain a numerical solution to the above boundary value problem, a suitable first guess of λ_1, λ_2, λ_3 at $t = t'$ must be made. Examination of results found with an IBM 704 computer has shown that a suitable first guess for the λ's is

$$\lambda_1 = \frac{W_{02}}{2.8\,\dot{W}\left(V_e + V_p\right)} = \frac{W_0 - \dot{W}_F^T\,t_B}{2.8\,\dot{W}\left(V_e + V_p\right)}$$

$$\lambda_2 = -\frac{1}{2}\,\lambda_1^0$$

$$\lambda_3 = \frac{0.71}{V_e + V_p}$$

If this leads to a negative t', the machine program automatically selects another initial set of λ's from among a group of five "built-in" guesses. If all of these fail, the machine prints out "T is negative" and the problem is terminated at this stage of the calculation.

To clarify notation the λ guesses will have a numbered superscript starting with zero; the first guess of λ_1, λ_2, λ_3 at t = t' is λ_1^0, λ_2^0, λ_3^0.

Using the first guess for the λ's (λ_1^0, λ_2^0, λ_3^0) along with L_3, R_3, and U_3, start at t' and integrate back through the set of differential equations, by means of Runge-Kutta, to obtain $R(0) = R^0$, $U(0) = U^0$, and $L(0) = L^0$. Let λ_{11}, λ_{21}, and λ_{31} be the final values of these functions in the correct solution. The correct values of R, U, and L at t = 0 are $R(0) = R_0 = R_u$, $U(0) = U_0 = U_2$ and $L(0) = L_0 = L_2$. A systematic method for generating additional sets of λ's such that

$$R^i - R_0 \leq \epsilon_R$$

$$U^i - U_0 \leq \epsilon_U$$

$$L^i - L_0 \leq \epsilon_L$$

will now be given. In these equations i represents the ith set of λ's used and ϵ_R, ϵ_U, and ϵ_L are prescribed

errors in radius, radial velocity, and angular velocity.

We now relate the λ's at $t = t'$ to the values of L, U, and R at $t = 0$ by the equations

$$L^0 - L_0 = f_1(\lambda_1^0, \lambda_2^0, \lambda_3^0) - f_1(\lambda_{11}, \lambda_{21}, \lambda_{31}) \tag{46}$$

$$= \alpha_{11}(\lambda_1^0 - \lambda_{11}) + \alpha_{12}(\lambda_2^0 - \lambda_{21}) + \alpha_{13}(\lambda_3^0 - \lambda_{31})$$

$$U^0 - U_0 = f_2(\lambda_1^0, \lambda_2^0, \lambda_3^0) - f_2(\lambda_{11}, \lambda_{21}, \lambda_{31}) \tag{47}$$

$$= \alpha_{21}(\lambda_1^0 - \lambda_{11}) + \alpha_{22}(\lambda_2^0 - \lambda_{21}) + \alpha_{23}(\lambda_3^0 - \lambda_{31})$$

$$R^0 - R_0 = f_3(\lambda_1^0, \lambda_2^0, \lambda_3^0) - f_3(\lambda_{11}, \lambda_{21}, \lambda_{31}) \tag{48}$$

$$= \alpha_{31}(\lambda_1^0 - \lambda_{11}) + \alpha_{32}(\lambda_2^0 - \lambda_{21}) + \alpha_{33}(\lambda_3^0 - \lambda_{31})$$

where the right hand side of these equations represents the first order terms of a Taylor series expansion of $f_1(\lambda_1^0, \lambda_2^0, \lambda_3^0)$ about $f_1(\lambda_{11}, \lambda_{21}, \lambda_{31})$ and so forth.

Now we continue to generate sets of λ's, subject to the condition that

$$t' = \left(\frac{W_{02}}{\dot{W}}\right) - g_0 I_{sp} \left[(\lambda_1^0)^2 + (\lambda_2^0)^2\right]^{1/2}$$

until

$$f_1(\lambda_1^i, \lambda_2^i, \lambda_3^i) - f_1(\lambda_{11}, \lambda_{21}, \lambda_{31}) = 0 \tag{49}$$

$$f_2(\lambda_1^i, \lambda_2^i, \lambda_3^i) - f_2(\lambda_{11}, \lambda_{21}, \lambda_{31}) = 0 \qquad (50)$$

$$f_3(\lambda_1^i, \lambda_2^i, \lambda_3^i) - f_3(\lambda_{11}, \lambda_{21}, \lambda_{31}) = 0 \qquad (51)$$

The procedure used to satisfy Eqs. 49, 50, and 51, was the following. Equations 46, 47, 48 are first re-written as

$$L^0 - L_0 = \lambda_{11}h_1^1 + \alpha_{12}h_2^1 + \alpha_{13}h_3^1 \qquad (52)$$

$$U^0 - U_0 = \alpha_{21}h_1^1 + \alpha_{22}h_2^1 + \alpha_{23}h_3^1 \qquad (53)$$

$$R^0 - R_0 = \alpha_{31}h_1^1 + \alpha_{32}h_2^1 + \alpha_{33}h_3^1 \qquad (54)$$

where

$$\lambda_1^0 - \lambda_{11} = h_1^1$$

$$\lambda_2^0 - \lambda_{21} = h_2^1$$

$$\lambda_3^0 - \lambda_{31} = h_3^1$$

In Eqs. 52, 53, 54, the quantities h_1^1, h_2^1, h_3^1 represent a correction in the choice of the first set of λ's, namely, λ_1^0, λ_2^0, λ_3^0. The second set of λ's chosen is

$$\lambda_1^1 = \lambda_1^0 - h_1^1 \tag{55}$$

$$\lambda_2^1 = \lambda_2^0 - h_2^1 \tag{56}$$

$$\lambda_3^1 = \lambda_3^0 - h_3^1 \tag{57}$$

and can be determined once h_1^1, h_2^1, h_3^1 are known. The values
for h_1^1, h_2^1, h_3^1 are determined from Eqs. 52, 53, 54 once the α
coefficients are known. The method for determining the
coefficients in Eqs. 52, 53, 54 is as follows.

Change λ_1^0 slightly to $\lambda_1^0 - \Delta\lambda_1$, and use this new set
of λ's, namely, $\lambda_1^0 - \Delta\lambda_1$, λ_2^0, λ_3^0, to obtain the time of
integration (t') from Eq. 45 and then integrate through the
differential equations. This yields

$$L^0 + \Delta L - L_0 = -\alpha_{11}\Delta\lambda_1 + \alpha_{11}(\lambda_1^0 - \lambda_{11}) \tag{58}$$
$$+ \alpha_{12}(\lambda_2^0 - \lambda_{21}) + \alpha_{13}(\lambda_3^0 - \lambda_{31})$$

$$U^0 + \Delta U - U_0 = -\alpha_{21}\Delta\lambda_1 + \alpha_{21}(\lambda_1^0 - \lambda_{11}) \tag{59}$$
$$+ \alpha_{22}(\lambda_2^0 - \lambda_{21}) + \alpha_{23}(\lambda_3^0 - \lambda_{31})$$

$$R^0 + \Delta R - R_0 = -\alpha_{31}\Delta\lambda_1 + \alpha_{31}(\lambda_1^0 - \lambda_{11}) \tag{60}$$
$$+ \alpha_{32}(\lambda_2^0 - \lambda_{21}) + \alpha_{33}(\lambda_3^0 - \lambda_{31})$$

Subtracting Eqs. 46, 47, 48 from Eqs. 58, 59, 60 yields

$$\alpha_{11} = - \Delta L / \Delta \lambda_1$$

$$\alpha_{21} = - \Delta U / \Delta \lambda_1$$

$$\alpha_{31} = - \Delta R / \Delta \lambda_1$$

The other coefficients are obtained by varying λ_2^0 and λ_3^0. The new set of λ's, namely, λ_1^1, λ_2^1, λ_3^1, are used to obtain a new t' from Eq. 45. With this set of λ's we integrate through our differential equations, starting at t = t' and integrating to t = 0, obtaining a new set of f's which are

$$f_1(\lambda_1^1, \lambda_2^1, \lambda_3^1) = L^1$$

$$f_2(\lambda_1^1, \lambda_2^1, \lambda_3^1) = U^1$$

$$f_3(\lambda_1^1, \lambda_2^1, \lambda_3^1) = R^1$$

Starting with this set of f's, the routine given above is used to obtain the new set of α coefficients. In obtaining these the same set of $\Delta\lambda$'s is used. Then a new set of h's is generated, which are h_1^2, h_2^2, h_3^2. From this set of h's, a new set of λ's is constructed, which is

$$\lambda_1^2 = \lambda_1^1 - h_1^2$$

$$\lambda_2^2 = \lambda_2^1 - h_2^2$$

$$\lambda_3^2 = \lambda_3^1 - h_3^2$$

This procedure is continued until

$$R^i - R_0 \leq \epsilon_R \tag{61}$$

$$U^i - U_0 \leq \epsilon_U \tag{62}$$

$$L^i - L_0 \leq \epsilon_L \tag{63}$$

where ϵ_R, ϵ_U, and ϵ_L are prescribed errors in radius, radial velocity, and angular velocity.

At this point the optimization equations were combined with the gravity-turn equations and coded for an IBM 704 computer. Details of the code are given in the appendix.

C. Results

1. Hydrogen system. The results for single-stage missiles having launch weights of 413,000 and 72,000 lb will now be given. In these results it is assumed that, for temperatures >1500°C, one is flying a nuclear rocket which has a corroding reactor or that the reactor has graphite-loaded fuel elements which are protected with niobium carbide. (See following section.)

41

Figures 4 through 9 give the final results obtained for a missile having a launch weight of 413,000 lb. Figure 4 gives circular orbit altitude as a function of reactor weight plus payload with the temperature as a parameter. To convert the results given in Fig. 4 in order to obtain circular orbit altitudes as a function of payload with the temperature as a parameter requires reactor weights which can only come from a detailed engineering study. Since no detailed engineering data exist for reactors having power densities of 0.5 Mw/lb and associated temperatures in excess of 1500°C, we assume that reactor weight can be expressed as (see page 290 of Ref. 3)

$$W_R = 10,000 + 2000 \frac{P_R}{T_c} \tag{64}$$

where

W_R = the weight of the reactor plus pressure shell

P_R = the reactor power in Mw

T_c = the gas temperature at the exit of the reactor in °C

Equation 64 is only used in connection with missions which have a vehicle launch weight of 413,000 lb. These missions require reactor powers which vary from 9000 Mw to 16,000 Mw and exit gas temperatures in excess of 1500°C. Equation 64
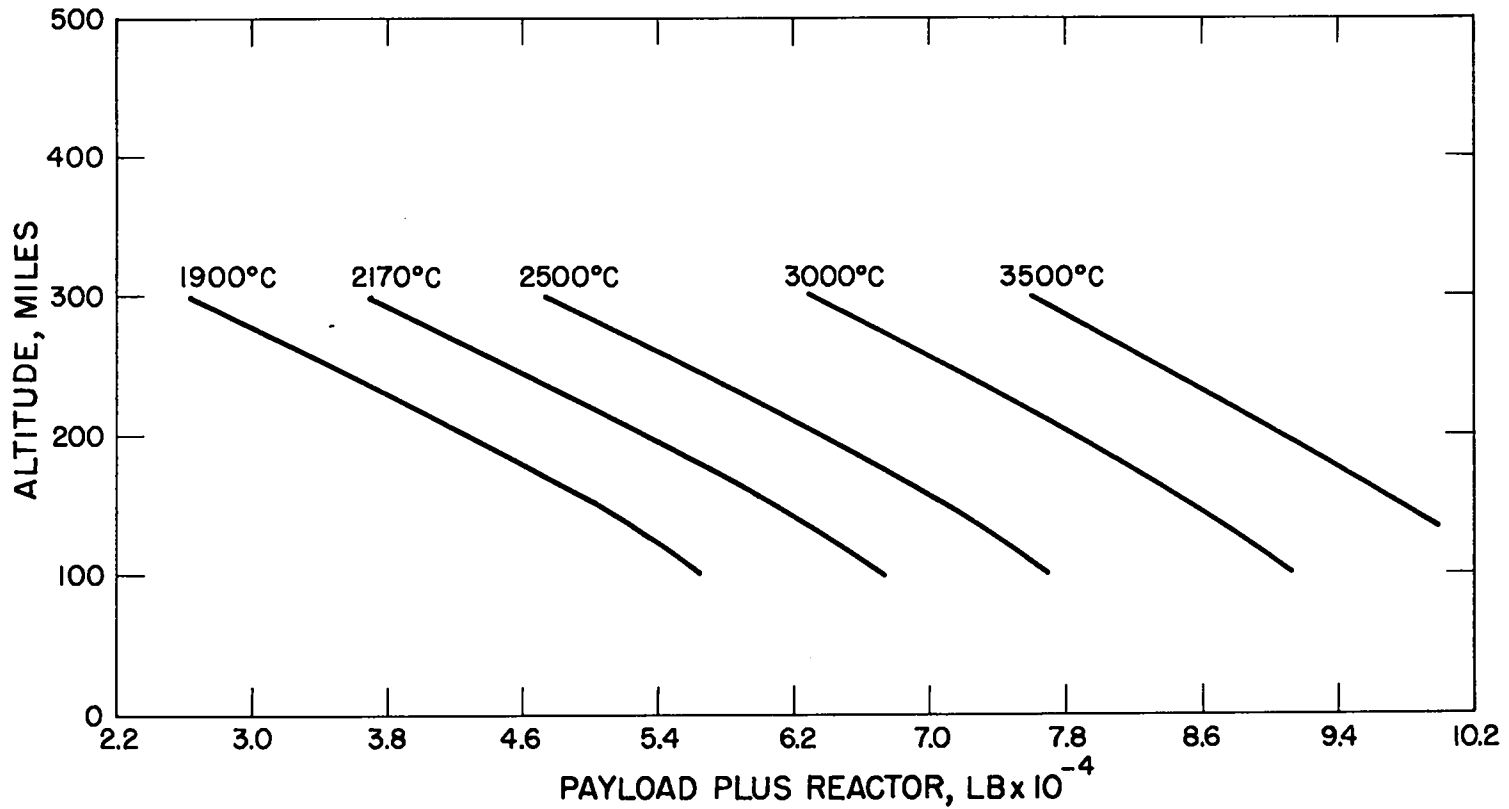
Fig. 4   Circular orbit altitude as a function of payload plus reactor weight
with the temperature taken as a parameter (results for PAW trajectories)
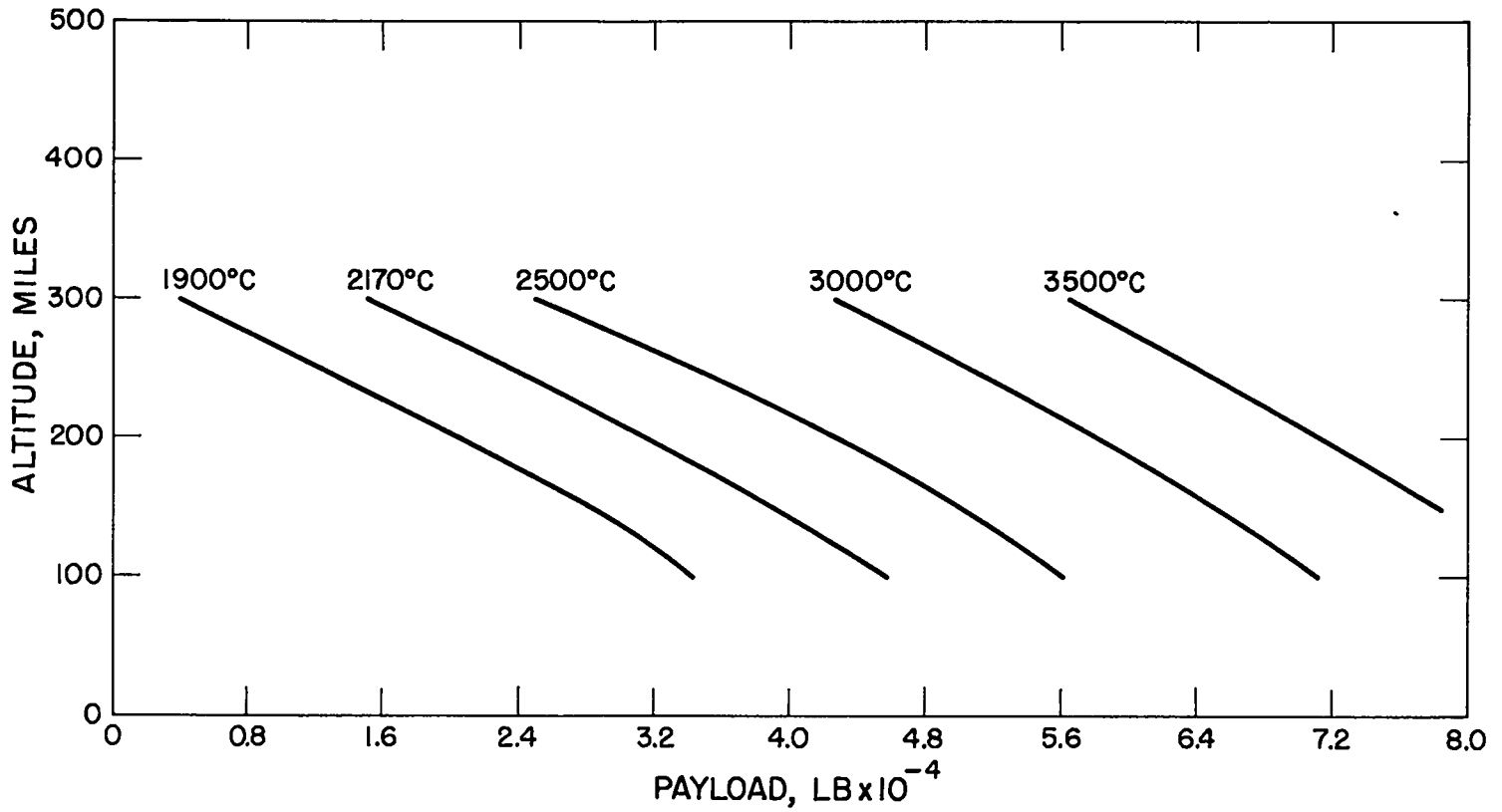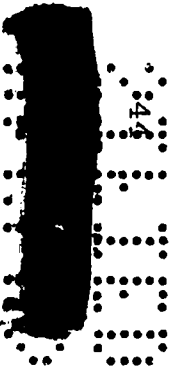
Fig. 5   Circular orbit altitude as a function of payload with the temperature
taken as a parameter (results for PAW trajectories)
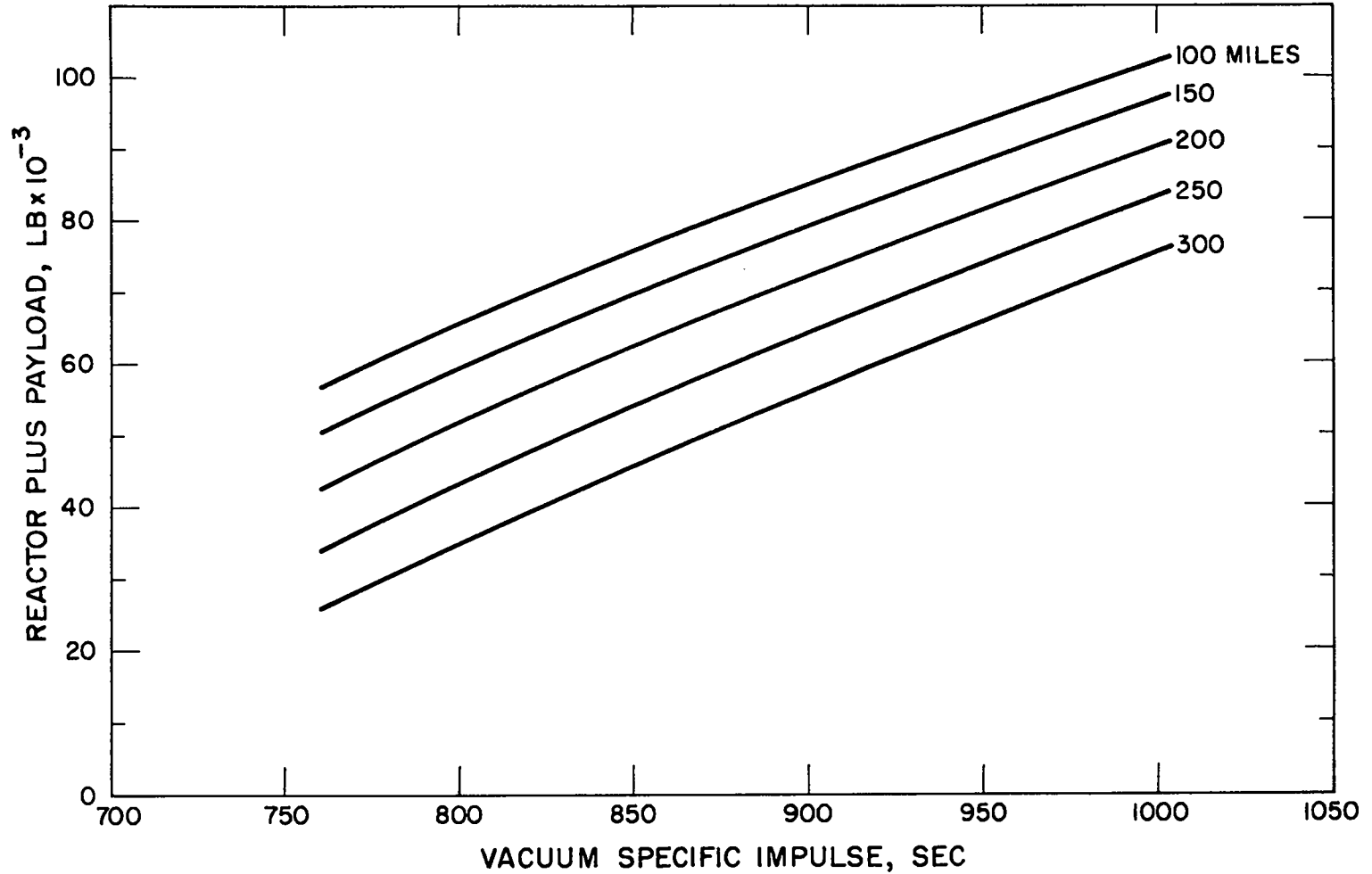
Fig. 6   Reactor plus payload in a given circular orbit as a function of vacuum
specific impulse with circular orbit altitude taken as a parameter
(results for PAW trajectories)
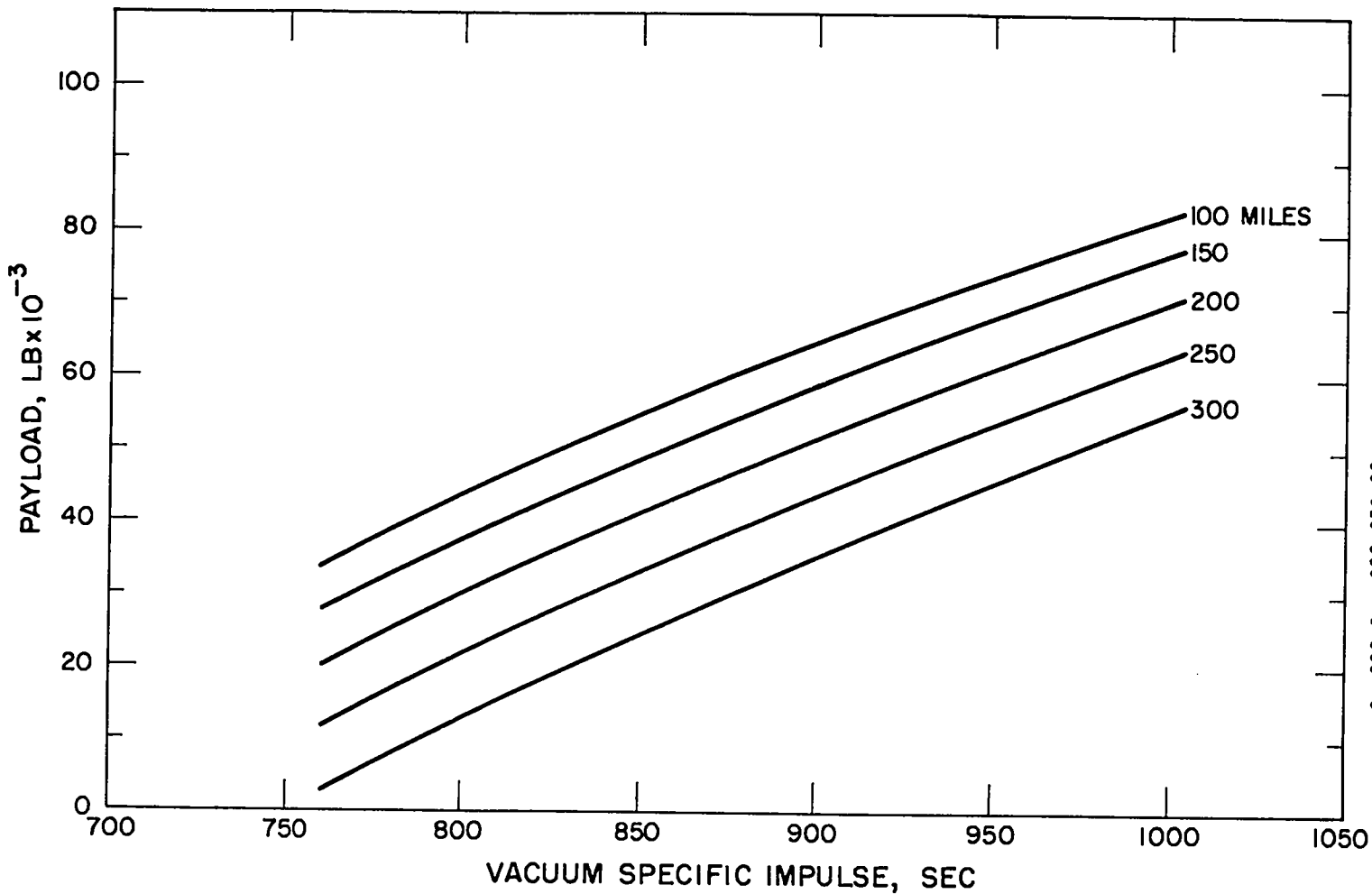
Fig. 7   Payload in circular orbit as a function of vacuum specific impulse with
circular orbit altitude taken as a parameter (results for PAW
trajectories)
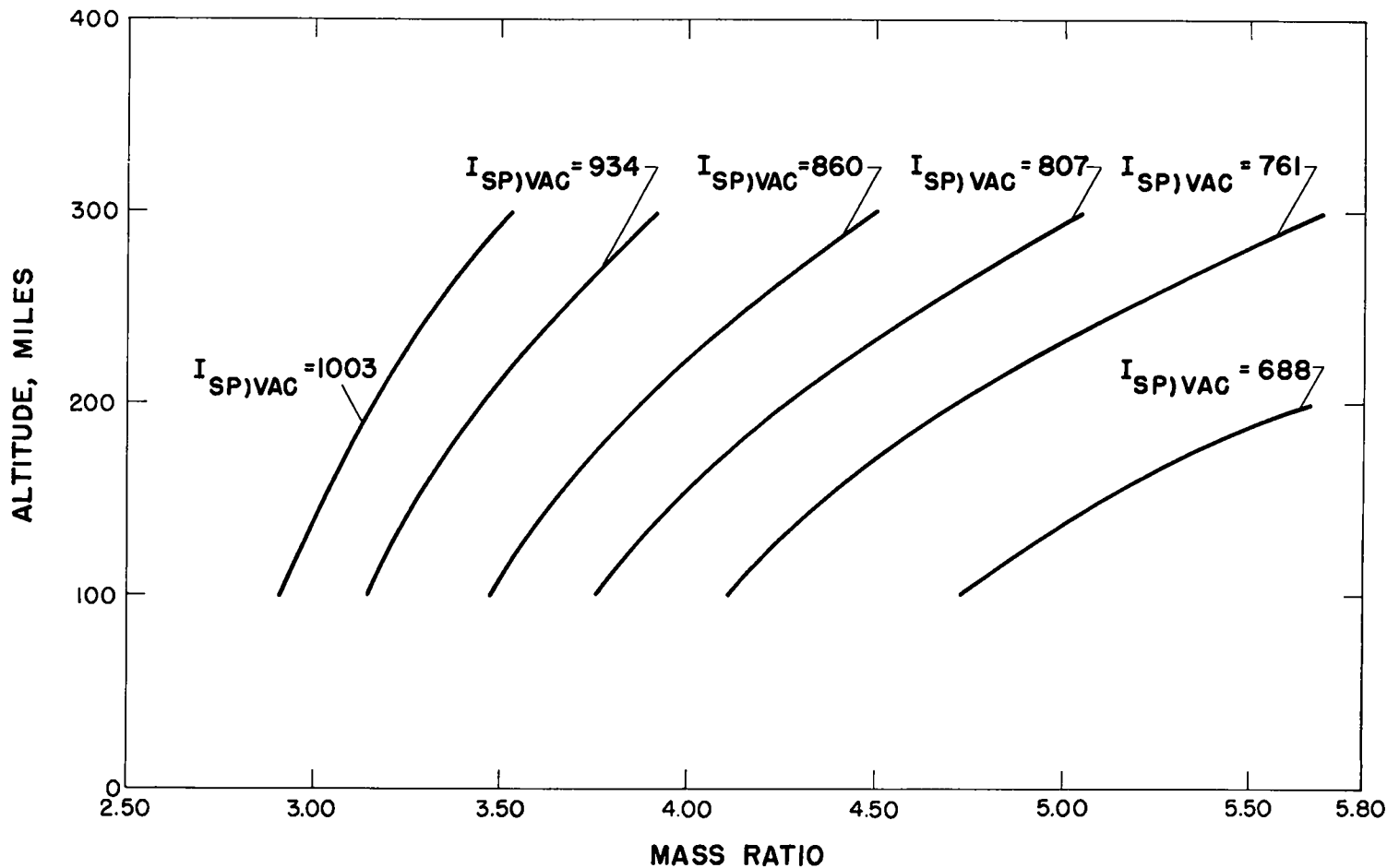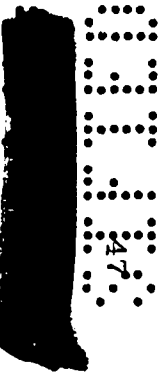
Fig. 8 Circular orbit altitude in miles as a function of mass ratio with the vacuum specific impulse taken as a parameter (results for PAW trajectories)
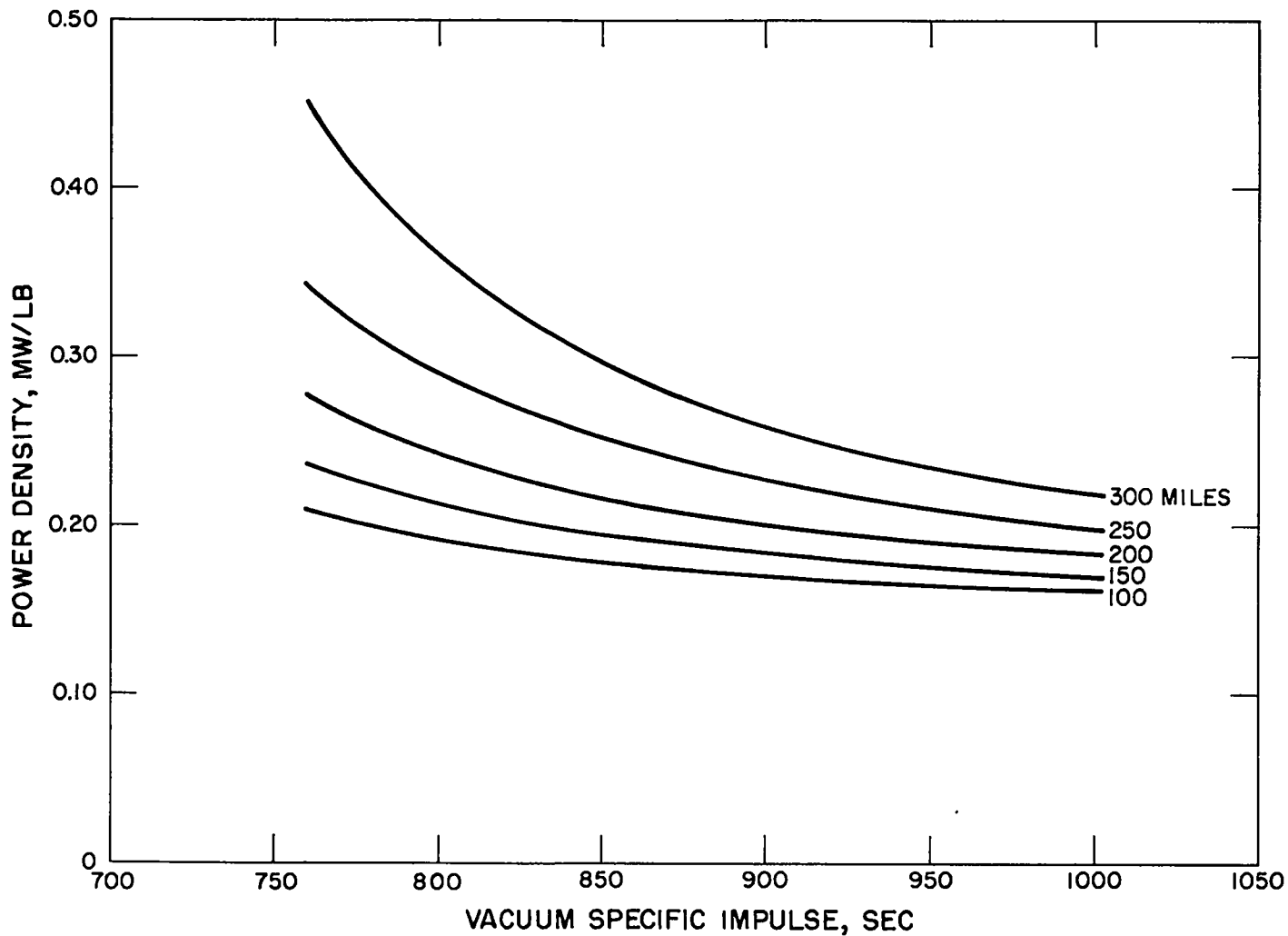
Fig. 9 Power density as a function of vacuum specific impulse with the circular orbit altitude taken as a parameter (results for PAW trajectories)

is used to obtain the results given in Fig. 5.

The only high power density, high temperature reactor for which a detailed engineering design exists, using hydrogen as the propellant, is the Condor reactor.[9] The weight of a Condor reactor, operating at a temperature of 1500°C and at a power level of 9000 Mw, is 22,800 lb. This represents a power density of $\sim$0.4 Mw/lb at a temperature of 1500°C. Substituting the Condor values for P_R and T_c into Eq. 64 yields a W_R of 22,000 lb; thus Eq. 64 is essentially anchored to the Condor results at 1500°C. Whether or not the reactor weights given by Eq. 64, for power densities greater than 0.4 Mw/lb and associated temperatures in excess of 1500°C, are possible with present day technology depends on forthcoming detailed engineering studies.

Figure 6 gives the reactor weight plus payload as a function of vacuum specific impulse with the circular orbit altitude taken as a parameter. The relationship between the gas temperature, in degrees centigrade, at the exit of the reactor and the vacuum specific impulse is given in Table 2.

TABLE 2

Effect of Temperature on Vacuum Specific Impulse

T_c, °C	$I_{sp})vac$
1900	761
2170	807
2500	860
3000	934
3500	1003

The results given in Fig. 7 were obtained from Fig. 6 by making use of the previous relationship, Eq. 64, for reactor weight. Figure 8 gives circular orbit altitude as a function of mass ratio (launch weight/burnout weight) with the vacuum specific impulse taken as a parameter. Figure 9 is a plot of power density as a function of vacuum specific impulse with the circular orbit taken as a parameter. The power density[10] is defined as the ratio of total jet power to the weight of the core, reflector, pressure shell, controls, and payload.

Figure 10 gives the results for a vehicle having a launch weight of 72,000 lb. The component weight break-down for this vehicle is given in Table 3. Also included in Fig. 10 is a curve which demonstrates the gains to be made by tank staging. No attempt was made to do an optimum
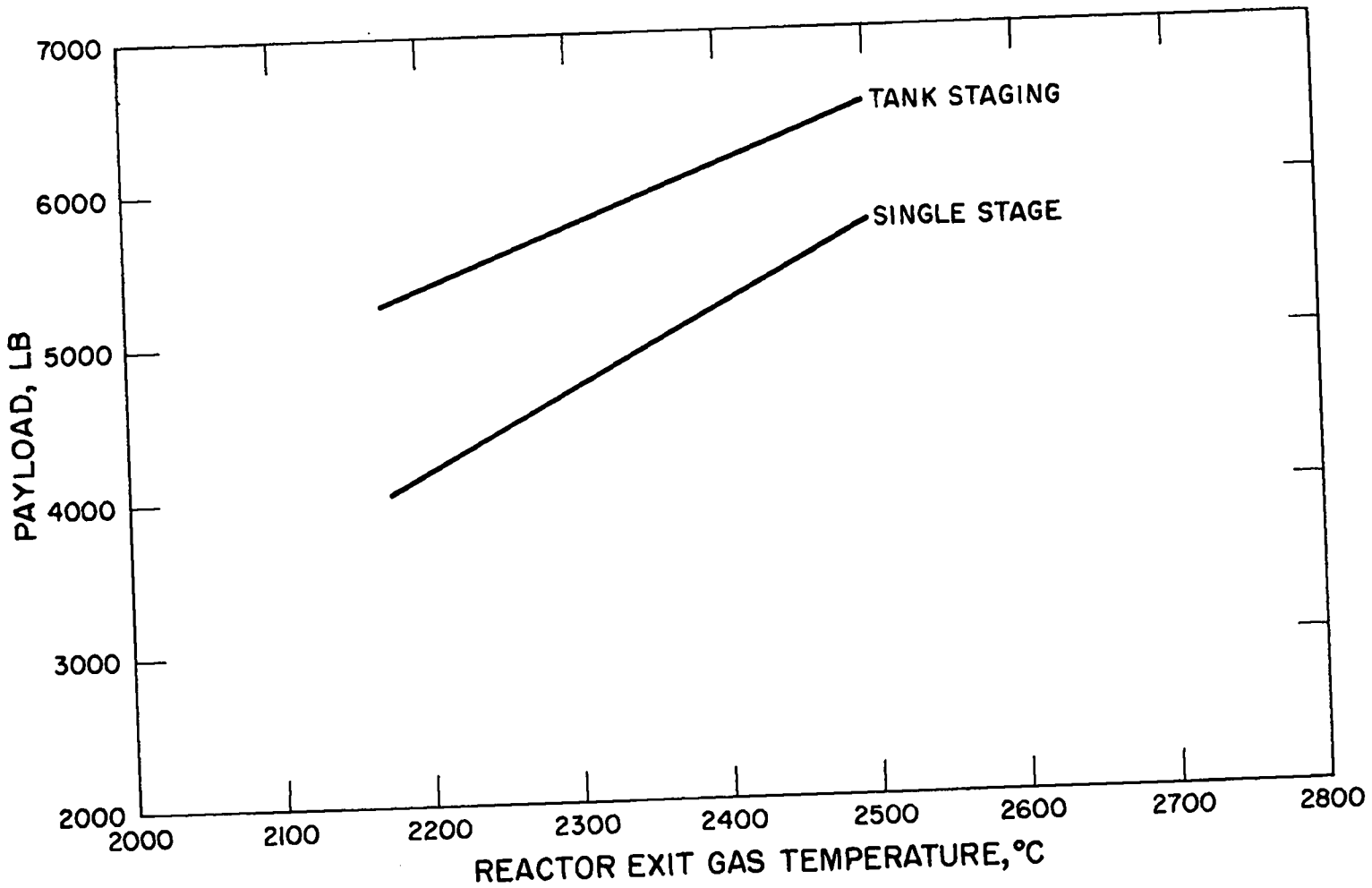
Fig. 10   Calculations showing the effect of tank staging on payload, placed in a 100-mi orbit, as a function of reactor exit gas temperature (results for PAW trajectories; launch weight used in calculations was 72,000 lb)

TABLE 3

Circular Orbit Mission Results for a Vehicle Having a
Launch Weight of 72,000 Lb

Launch weight	72,000 lb
Flow rate	141.5 lb/sec
Reactor power	2214 Mw
Reactor exit gas temp	2170°C
Pump and turbine weight	700 lb
Nozzle weight	400 lb
Structure weight	1000 lb
Tank weight	4200 lb
Reactor weight	8000 lb
Fuel weight	53,100 lb
Holdover weight	600 lb
Tank diameter	14.5 ft
Ratio of over-all length to diameter	6
Payload weight	4000 lb

tank-staging calculation. The calculations were done by
merely dropping a tank when the rocket reached an altitude
of 200,000 ft. The weight of the tankage dropped was
equivalent to the tank weight required to contain the
fuel expended in reaching staging altitude. These
calculations were performed to demonstrate that tank
staging, if mechanically feasible, is quite advantageous.

2. <u>Hydrogen-methane system</u>. Experimentally it has
been shown that graphite, at the temperatures of interest
in nuclear rocket propulsion, is subject to severe
corrosion by hydrogenous working fluids; and in order to
achieve a useful long life of even the few minutes required
by a rocket engine, some means of protecting the graphite
surface is required.

Several approaches to this problem have been studied
at LASL. One approach is to render the working fluid
neutral towards hot graphite by the addition of some
carbon-containing material such as methane, ethane, etc.,
to the hydrogenous working fluid. Another approach, which
is showing promise, is to coat or line the surface of the
graphite with niobium carbide. Results indicate that a
working fluid of hydrogen and methane (3 mole %) at a
temperature of 2100°C is possible without graphite
corrosion.[11] Without the methane the temperature of the

working fluid, for no graphite corrosion, is $\sim 1500°C.$[12]
Preliminary experiments on coating (or lining) the graphite
indicate the possibility of a hydrogen working fluid
temperature of $\sim 2500°C.$[13]

The technology of protecting the graphite-loaded fuel
elements with niobium carbide has advanced to the point
where the consideration of a working fluid of hydrogen-
methane is no longer attractive. An example of the
performance of a nuclear rocket using a working fluid of
hydrogen and methane (3 mole %) is given in Fig. 11 for
an exit gas temperature of 1900°C.

V. Elliptic Ascent

In these calculations the vehicle was flown from
launch pad to height h_2 by means of a gravity turn and
was then powered from h_2 into a 100-mi circular orbit in
a manner that optimized payload. For purposes of
discussion this 100-mi orbit will be referred to as orbit 1.
The choice of the perigee orbit at 100 mi was due to
practical considerations.[14] The vehicle was flown from
orbit 1 to various other circular orbits, all of which
were at altitudes greater than 100 mi. The equations used
for obtaining satellite payloads, for the elliptic ascent
trajectory, will now be given. For this discussion we
consider the flight of a vehicle between orbit 1 and
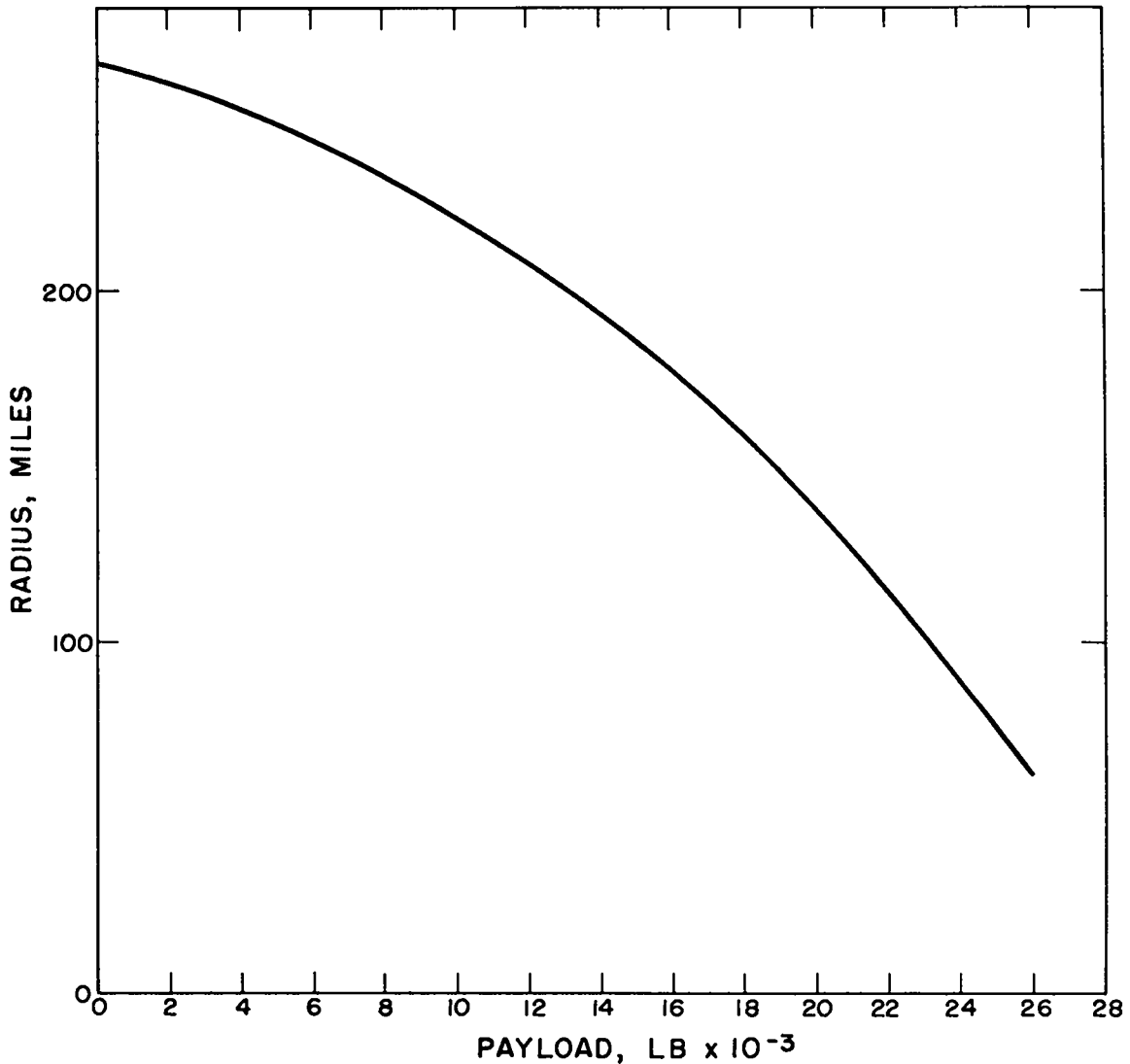
Fig. 11   Circular orbit radius as a function of payload for
a vehicle containing a working fluid of hydrogen
and methane (3 mole %) and operating at an exit
gas temperature of 1900°C (results for PAW
trajectories; launch weight used in calculations
was 413,000 lb)

orbit 2, where the radius of orbit 2 is greater than orbit 1. The velocity increment required to transfer the rocket from circular orbit 1 to an elliptic orbit, with apogee at circular orbit 2, is

$$\Delta V_1 = V_p - V_{c1} \tag{65}$$

where

V_p = the vehicle velocity at perigee in its elliptical orbit

V_{c1} = the vehicle velocity in circular orbit 1

The velocity increment required to transfer the vehicle from the apogee of the elliptic orbit into circular orbit 2 is

$$\Delta V_2 = V_{c2} - V_a \tag{66}$$

where

V_a = the vehicle velocity at apogee in its elliptical orbit

V_{c2} = the vehicle velocity in circular orbit 2

The equation for the velocity of a vehicle in a circular orbit is

$$V_c = \left(\frac{g_0 R_e^2}{R} \right)^{1/2} \tag{67}$$

56

where

R = the distance from the center of the earth to the vehicle (considered to be a mass point)

g_0 = 32.17 ft/sec^2

R_e = radius of the earth = 2.0908 x 10^7 ft

The equation for the velocity of a vehicle in an elliptic orbit is[15]

$$V^2 = 2g_0 R_e^2 \left(\frac{1}{R} - \frac{1}{R_1 + R_2} \right) \qquad (68)$$

where

R_1 = the radius of orbit 1

R_2 = the radius of orbit 2

The total ΔV required to go from orbit 1 to orbit 2, obtained by combining Eqs. 65 through 68, is

$$\Delta V = V_{c1} \left\{ x^{-1/2} \left[1 - 2^{1/2} (1 - x) (1 + x)^{-1/2} \right] - 1 \right\} \qquad (69)$$

where

$$x = \frac{\text{final orbit radius}}{\text{initial orbit radius}}$$

The burnout weight in orbit 2 can be obtained from a knowledge of the vehicle weight in orbit 1, the ΔV required for the transfer, and the vehicle vacuum I_{sp}. The equation relating these quantities is

57

$$\Delta V = g_0 I_{sp)vac} \ln R \qquad\qquad (70)$$

where

$$R = \frac{\text{weight in orbit 2}}{\text{weight in orbit 1}}$$

From a knowledge of the burnout weight in orbit 2 and the various component weights of the vehicle - tank weight, reactor weight, etc. - the payload in orbit 2 can be obtained. The results of this analysis are given in Table 4. The weight breakdown for flying flight 3 into a 300-mi orbit is given in Table 5.

In the Hohmann transfer part of the analysis it was assumed that the circular orbits were coplanar and that there were no gravitational losses.

VI. Ballistic Ascent

This trajectory was obtained from the previous work given in sections III and IV. The vehicle was flown straight up for a period of 16 sec, at which time it was kicked over to β_k and then flown to a specified height $(h_{c.o.})$. Upon reaching $h_{c.o.}$ The thrust was turned off and the vehicle allowed to coast to summit, $dh/dt = 0$, at which time the thrust was cut back in and the vehicle powered into a 300-mi orbit. The optimization equations given in section IV were used to calculate the portion of the flight from summit to the 300-mi orbit.

TABLE 4

Optimum Trajectory Mission Results

Flight	Temp, °C	$I_{sp)vac}$	W_R, lb	P_R, Mw	Launch wt, lb	Payload in Orbit, lb						
						100 mi	200 mi	300 mi	500 mi	1000 mi	5000 mi	∞
1	1500	688	24,000	10,312	415,000	17,100	15,800	14,400	12,100	7,200	-12,200	-19,400
2	1900	761	22,000	11,752	413,000	34,200	32,900	31,600	29,100	23,600	3,500	-4,300
3	2170	807	22,000	12,633	413,000	45,800	44,400	43,000	40,500	35,200	13,700	5,600
4	2500	860	21,000	13,669	413,000	56,100	54,700	53,200	50,800	45,300	23,300	14,800
5	3000	934	20,000	15,072	412,000	71,100	69,600	68,200	65,800	59,900	37,400	28,400
6	3500	1003	19,000	16,355	412,000	83,700	82,200	80,800	78,200	72,100	49,600	40,500
7	2170	807	8,000	2,214	72,000	4,000	3,800	3,500	3,100	2,200	-1,400	-2,800
8	2500	860	8,000	2,391	72,000	5,800	5,500	5,300	4,800	3,900	180	-1,300
9*	2170	807	8,000	2,214	72,000	5,300	5,100	4,900	4,500	3,600	190	-1,100
10*	2500	860	8,000	2,391	72,000	6,700	6,500	6,200	5,800	4,900	1,400	-10

*Tank dropped at 200,000 ft.

TABLE 5

Weight Breakdown for Flight No. 3

H_2 load	308,000 lb
H_2 tank	24,000 lb
Structure	4,000 lb
Turbopump	9,000 lb
Nozzle	3,000 lb
Reactor and pressure shell	22,000 lb
Payload	43,000 lb
Launch weight	413,000 lb
Tank diameter	25 ft
Ratio of over-all length to diameter	6
Flow rate	800 lb/sec
Reactor power	12,630 Mw
$I_{sp)vac}$	807 sec
Dry weight	62,000 lb

To determine the most efficient ballistic trajectory from the standpoint of largest payload in the 300-mi orbit, curves of total vehicle weight in orbit as a function of β_k were obtained with $h_{c.o.}$ as a parameter. From these curves a value for maximum total vehicle weight was determined for a given $h_{c.o.}$ and a curve of total vehicle weight (maximum) as a function of $h_{c.o.}$ was obtained. The results are given in Fig. 12. The vehicle launch weight for this problem was 413,000 lb and the reactor exit gas temperature used was 2170°C. These are the same values used for flight No. 3 of Table 4. A comparison of the payload for the ballistic ascent (41,000 lb) with that of the elliptic ascent (43,000 lb) shows that, for a 300-mi circular orbit, the elliptic ascent is slightly superior to the ballistic ascent.

VII. Conclusions

The trajectory calculations demonstrate quite clearly that, for orbits above 100 mi, the ballistic and elliptic ascents are far superior to the PAW ascent and that the elliptic ascent is superior to the ballistic ascent. In the elliptic and ballistic ascent calculations it was assumed that the thrust could be turned off and on during flight. This demonstrates the gains to be made by the use of a nuclear rocket motor which can be recycled.
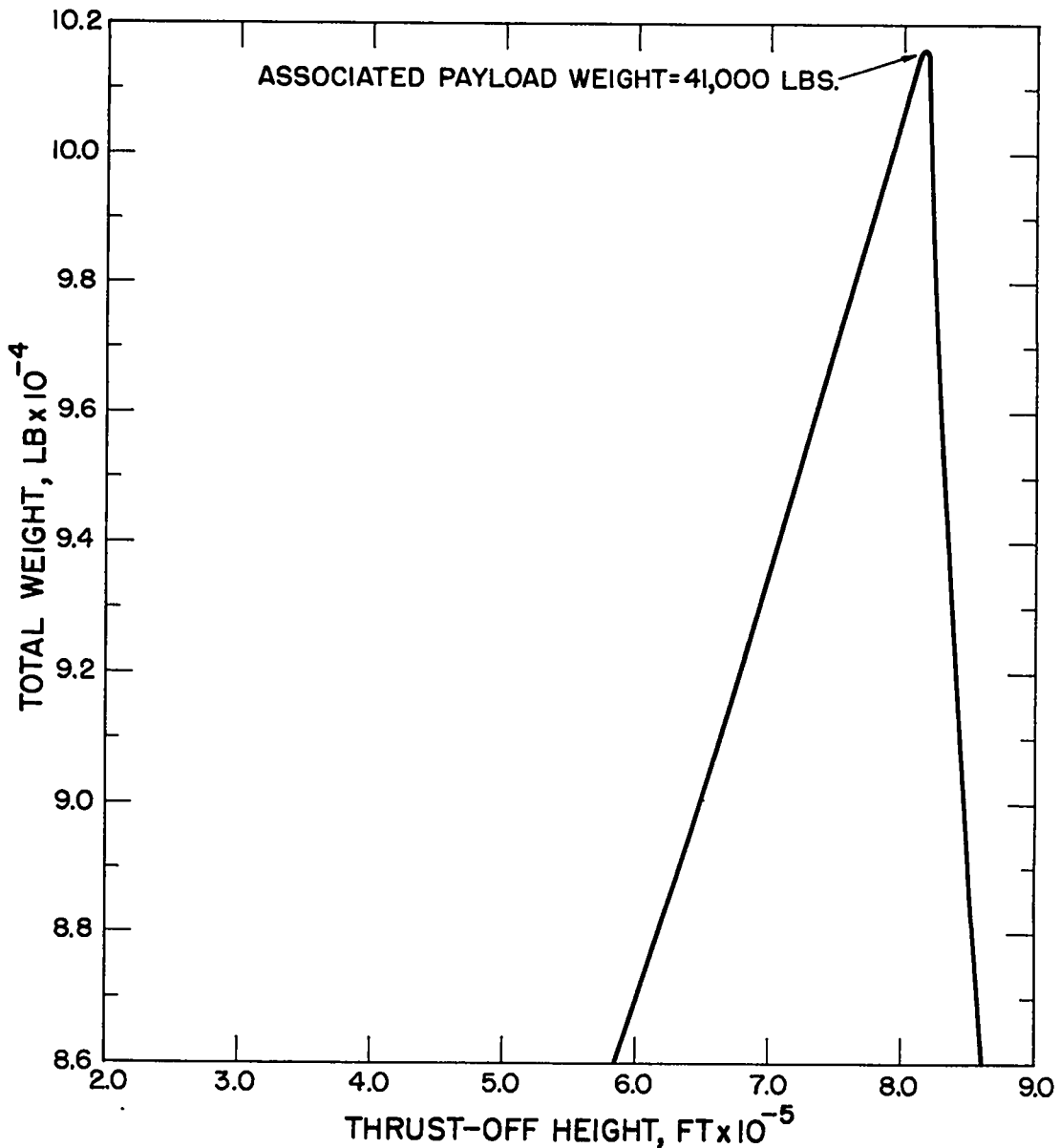
Fig. 12 Total weight in pounds, placed in a 300-mi orbit
as a function of thrustoff height in feet for a
vehicle exit gas temperature of 2170°C, and a
launch weight of 413,000 lb (results for a
composite trajectory consisting of a gravity-turn
part, ballistic trajectory to apogee, and then
along an optimum path into a 300-mi orbit; results
shown here are for an optimum value of β_k)

Another way of pointing out the effect the trajectory has on a given mission is in terms of power density. The power density requirement at 1900°C (see Fig. 9) for a 300-mi circular orbit mission drops from 0.452 Mw/lb to 0.208 Mw/lb in going from a PAW ascent to an elliptic ascent. The sensitivity of payload in orbit to temperature is also reduced when a vehicle is flown along an elliptic ascent trajectory into a low earth orbit (300 mi).

In section V it was shown that a single-stage vehicle having a launch weight of 413,000 lb and operating at a temperature of 2170°C could place 43,000 lb of payload into a 300-mi orbit. The three-stage Saturn-Titan-Centaur vehicle, having a launch weight of 1,075,000 lb,[16] will place 25,000 lb into a 300-mi orbit. Replacing the Titan with a LOX-Hydrogen system will increase the payload to approximately 30,000 lb. The dry weight of the nuclear vehicle is 62,000 lb as compared to approximately 90,000 lb for the Saturn-Titan-Centaur vehicle.

A nuclear system having a launch weight of 72,000 lb, reactor power of 2214 Mw, and operating at an exit gas temperature of 2170°C (close to present day technology) will place 3500 lb of payload into a 300-mi orbit. Tank staging, see section V, will increase the payload to 4900 lb. The Atlas-Centaur plus a small solid-propellant

third stage will place approximately 7400 lb of payload into a 300-mi orbit. The dry weights of the nuclear and chemical systems are approximately the same.

Caution must be employed in drawing conclusions from the performances of the nuclear and chemical vehicles described above. The reason is that the performance of the chemical vehicle is based on multiple staging, whereas all the data presented for the nuclear systems are based on a single stage. An example of this point is that by going to a two-stage nuclear vehicle with a launch weight of 413,000 lb the payload can be increased from 43,000 lb to better than 70,000 lb. Circular orbit missions for two-stage nuclear systems will be given in a later report. The gains to be made in staging must also be weighed against a decrease in over-all reliability.

From the above performance data, but realizing the implications of comparing a three-stage chemical rocket with a single-stage nuclear rocket, one concludes that nuclear systems are superior to their chemical counterpart (same dry weight) for missions which require large payloads in orbit. The reverse is true for Atlas-Centaur type missions (small payloads). The reason for this is that the reactor weight does not decrease substantially with lowering power requirements; thus the power density

decreases, which in turn causes the percentage of dry
weight which can go into payload to decrease. Future
technology could very well lead to lower reactor weights,
lower tank weights, and exit gas temperatures greater than
2170°C, thus leading to the increased performance of
small-launch-weight nuclear systems. Present day technology
points in this direction.

REFERENCES

1. K. W. Ford, Internal memoranda.

2. W. Hohmann, Die Erreichbarkeit der Himmelskorper,
   Druck und Verlag R. Oldenbourg, Munich and Berlin,
   1925.

3. Los Alamos Scientific Laboratory Report LAMS-2217
   (Vol. II), March 20, 1958.

4. L. Turner, Internal memorandum.

5. R. L. Aamodt et al., Los Alamos Scientific Laboratory
   Report LA-1870, March 1955, p. 53.

6. J. L. Hartman, Internal memorandum.

7. G. P. Sutton, Rocket Propulsion Elements, John Wiley
   and Sons, Inc., New York, 1956.

8. G. A. Bliss, Calculus of Variations, University of
   Chicago Press, Chicago, 1925, p. 202.

9. F. P. Durham and V. L. Zeigner, Internal memorandum.

10. G. M. Grover and E. W. Salmi, Internal memorandum.

11. H. Filip and A. A. Gonzalez, Internal document.

12. H. Filip and A. A. Gonzalez, Internal document.

13. C. E. Landahl, Internal document.

14. K. A. Ehricke, Astronaut. Acta, 4 Vol. II, 175, 1956.

15. L. Page, Introduction to Theoretical Physics (2nd ed.), D. Van Nostrand Company, New York, 1947 p. 95.

16. ABMA-DIR-TR-2-59, June 1959, p. 23.
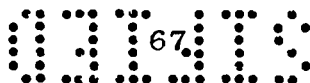
APPENDIX

FLOW DIAGRAM AND NUMERICAL METHOD

Input, box 1. Input numbers and identification are
entered into the machine on six cards as indicated. On
the first card any desired information (such as date,
name of problem, etc.) may be punched in columns 2 through
72 inclusive. On the other five cards, numbers are punched,
seven per card, in floating point form with ten columns
per number. Each number has the format $\pm$x.xxxxx$\pm$xx. The
decimal point is not punched but is assumed to be in the
position shown. The last two digits along with the sign
in the third last position are the power of 10 by which
the first six digits (with point as shown) are multiplied.
The nth number is punched in columns $10(n - 1) + 1$ through
$10n$ inclusive, with the sign of the number in column
$10(n - 1) + 1$ and the sign of the exponent of 10 in column
$10(n - 1) + 8$.

Output. Output consists of a printed listing as shown
in boxes 2, 4, and 13. Since the printer has only upper-
case letters and no superscripts or subscripts, the table
below is given to show the correspondence between the
printing on the listing and the symbols used in the report
and the flow diagram.

On Listing	In Report
Box 2:	
BETAKD	β_k (printed in degrees)
CAPK	K
AO	a_O
BETAB	$\bar{\beta}$
TC	T_c
FK	k
CNL	C_{NL}
AEAT	A_e/A_t
PC	P_c
FD	f_D
WFO	W_F^T
RHOT	ρ_T
RHOH2	ρ_{H_2}
RHOCH4	ρ_{CH_4}
SMA	a
FN1BAR	$\bar{n}_1$
FN2BAR	$\bar{n}_2$
ABAR	$\bar{A}$
CPH2	C_p
PD	P_d
CN	C_N
WL	W_L

68

WR	W_R
FM1	M_1
FM2	M_2
VZ	V_0
TK	t_k
DELTI	$\triangle t$
ATCOEF	A_T
FM	M_{av}
THOFFH	$h_{c.o.}$
THONH	$h_{c.on}$
H2F	h_2
HF	h_f
SETM4	SETM4

Box 4:

WH2	$W_F^{H_2}$
WDOTH2	$\dot{W}_F^{H_2}$
WTH2	$W_T^{H_2}$
WSH2	$W_S^{H_2}$
WTP	W_P^T
WO	W_0
WCH4	$W_F^{CH_4}$
WDOTCH	$\dot{W}_F^{CH_4}$
WTCH4	$W_T^{CH_4}$
WSCH4	$W_S^{CH_4}$

WN	W_N
WDOT	$\dot{W}^T_F$
F	F
DT	D_t
T	t
V	V
BETAD	β (printed in degrees)
H	h
THETAD	θ (printed in degrees)
TANACCEL	$\dot{V}$
BETAPD	$\dot{β}$ (printed in degrees/sec)
HP	$\dot{h}$
THETAPD	$\dot{θ}$ (printed in degrees/sec)
L	L
THRUST	T
ISP	I_{sp}

Box 13:

T	t
L	L
U	U
H	h
LAM1	$λ_1$
LAM2	$λ_2$
LAM3	$λ_3$
W(T)	W(t)
TH/W	T/W(t)
V	V

SIFTED

ACCEL	ACCEL
THETAD	θ (printed in degrees)
THETDD	$\dot{\theta}$ (printed in degrees/sec)
BETAD	β (printed in degrees)
GAMMAD	γ (printed in degrees)

Options. The various ways of running the problem are determined by the five input parameters W_F^T, $h_{c.o.}$, $h_{c.on}$, h_2, and SETM4. These variations center on when and how the thrust is turned off and on. The thrust is turned off either by having the missile run out of fuel (determined by W_F^T) or by specifying a cut-off height, $h_{c.o.}$. It is turned on again either at the cut-on height, $h_{c.on}$, or at the time when $\dot{h} = 0$, depending on the value of SETM4. If SETM4 $\neq$ 0, the thrust is turned on when $h \geq h_{c.on}$, and if SETM4 = 0, the thrust is turned on when $\dot{h} \leq 0$. The missile goes under control of the variational part when it reaches height h_2, provided the thrust is on.

The tabulation shows various possibilities. Here "normal" means that the quantity referred to is of a size which can be expected; for example, "normal" for $h_{c.o.}$ would be a value of h at which it is reasonable to turn off the thrust, whereas "large" means larger than h can ever attain. "Immaterial" means that the corresponding quantity is never consulted.

UNCLAS 73

$h_{c.o.}$	$h_{c.on}$	h_2	SETM4	Remarks
large	immaterial	large	immaterial	Missile runs out of fuel and finally hits ground.
large	immaterial	normal	immaterial	Missile goes up and into variational part when $h \geq h_2$ with thrust on all the time.
normal	large	large	$\neq 0$	Thrust is turned off when $h \geq h_{c.o.}$, and missile finally hits ground.
normal	normal	normal	$\neq 0$	Thrust is turned off when $h \geq h_{c.o.}$, is turned back on when $h \geq h_{c.on}$, and missile goes into variational part when $h \geq h_2$.
normal	immaterial	normal	0	Thrust is turned off when $h \geq h_{c.o.}$, is turned back on when $h = 0$, and missile goes into variational part when $h \geq h_2$.
normal	normal	large	$\neq 0$	Thrust is turned off when $h \geq h_{c.o.}$, is turned back on when $h \geq h_{c.on}$, and missile eventually runs out of fuel and finally hits ground.
normal	immaterial	large	0	Thrust is turned off when $h \geq h_{c.o.}$, is turned back on when $h = 0$, and missile eventually runs out of fuel and finally hits ground.

Start — From Boxes 13, 26, 27

Box 1

Read input:

- 1st card: Identification
- 2nd card: β_k K a_0 $\bar{\beta}$ T_o k C_{NL}
- 3rd card: A_e/A_t P_c t_D W_F^T ρ_T ρ_{H_2} ρ_{CH_4}
- 4th card: a $\bar{n}_1$ $\bar{n}_2$ $\bar{A}$ C_p P_d C_N
- 5th card: W_L W_R M_1 M_2 V_0 t_k Δt
- 6th card: AT M_{av} $h_{o.o.}$ $h_{c.on}$ h_2 h_f SETM4

Box 2

Print:

Bill Anderson 7-4560   Deck of Mar. 28, 1960
Identification (Date, Name of Problem, etc.)

BETAKD=	CAPK=	AO=	BETAB=	TC=	FK=
CNL=	AEAT=	PC=	FD=	WFO=	RHOT=
RHOH2=	RHOCH4=	SMA=	FN1BAR=	FN2BAR=	ABAR=
CPH2=	PD=	CN=	WL=	WR=	FM1=
FM2=	VZ=	TK=	DELTI=	ATCOEF=	FM=
THOFFH=	THONH=	H2F=	HF=	SETM4=	

Box 3

(Time independent quantities)

Note: $g_0 = 32.17$
$\bar{R} = 2782$

Solve for P_e: $\dfrac{A_e}{A_t} = \left(\dfrac{2}{k+1}\right)^{1/(k-1)}\left(\dfrac{P_e}{P_c}\right)^{-1/k}\left(\dfrac{k-1}{k+1}\right)^{1/2}\left[1 - \left(\dfrac{P_e}{P_c}\right)^{(k-1)/k}\right]^{1/2}$

Find:

$$C^* = \sqrt{\dfrac{g_0 \bar{R} T_c}{k\, M_{av}\left(\dfrac{2}{k+1}\right)^{(k+1)/(k-1)}}}$$

$$A = \begin{cases} 1 - \dfrac{14.7}{P_e} & \text{if } P_e \geq 4.9 \\ -2 & \text{if } P_e < 4.9 \end{cases}$$

(If $P_e < 4.9$, the remark "exit pressure is less than 4.9" appears on listing)

$$v_p^0 = A\,\dfrac{P_e}{P_c}\,\dfrac{A_e}{A_t}\,C^*$$

$$\eta = \sqrt{1 - \left(\dfrac{P_e}{P_c}\right)^{(k-1)/k}}$$

$$V_e = C_{NL}\,\eta\,\sqrt{\dfrac{2g_0 \bar{R} k T_c}{(k-1) M_{av}}}$$

$$F = \dfrac{\bar{n}_2 M_2}{\bar{n}_1 M_1 + \bar{n}_2 M_2}$$

$$\zeta = \dfrac{\bar{A}\,\rho_{H_2}}{\rho_{CH_4}} - 1$$

$$\phi = \dfrac{.000648\,(K+2)}{(1-a)\left(\dfrac{4}{3}+K\right)}$$

$$W_F^{H_2} = (1 - F)\,W_F^T$$

$$W_F^{CH_4} = W_F^T - W_F^{H_2}$$

$$W_T^{H_2} = \dfrac{\phi(1-F)\,W_F^T \rho_T}{\rho_{H_2}}$$

$$W_T^{CH_4} = \dfrac{\phi\,\bar{A}F\,W_F^T \rho_T}{\rho_{CH_4}}$$

$$W_S^{H_2} = .15\,W_T^{H_2}$$

$$W_S^{CH_4} = .15\,W_T^{CH_4}$$

$$S = \dfrac{(1-F)\,C_p\sqrt{P_d}}{\rho_{H_2}}$$

$$W_0 = \dfrac{W_L + W_R + \dfrac{W_F^T}{\rho_{H_2}}\left[1.15\,\rho_T\,\phi(F\zeta + 1) + \rho_{H_2}\right]}{1 - \dfrac{32.17\left(1 - a_0\right)}{(1-\beta)\left(v_e + v_p^0\right)}\left(S + \dfrac{C_N C^*}{32.17\,P_c}\right)}$$

$$\dot{W}_F^T = \dfrac{32.17\,W_0\left(1 + a_0\right)}{(1-\beta)\left(v_0 + v_p^0\right)}$$

$$\dot{W}_F^{H_2} = (1 - F)\,\dot{W}_F^T$$

$$\dot{W}_F^{CH_4} = \dot{W}_F^T - \dot{W}_F^{H_2}$$

$$w_p^T = S\,\dot{W}_F^T$$

$$W_N = \dfrac{C_N C^*\,\dot{W}_F^T}{32.17\,P_c}$$

$$D_T = \left[\dfrac{8(1-F)\,W_F^T}{\pi\,\rho_{H_2}\,(1-a)\left(\dfrac{4}{3}+K\right)}\right]^{1/3}$$

$$E = \dfrac{1}{8}\,g_0\,\pi\,f_D\,D_T^2$$

Set initial conditions for differential equations:

$V = V_0$ $h = 0$ $t = 0$

$\beta = 0$ $\theta = 0$

Set branch 1 to exit 1
Set branch 2 to exit 1
Set branch 3 to exit 1
Set branch 4 to exit $\begin{cases} 1 & \text{if SETM4} = 0 \\ 2 & \text{if SETM4} \neq 0 \end{cases}$

To Box 4

From Box 3

Box 4

Print:

WH2=	WDOTH2=	WTH2=	WSH2=	WTP=	WO=
WCH4=	WDOTCH=	WTCH4=	WSCH4=	WN=	WDOT=
F=	DT=				

T V BETAD H THETAD TANACCEL BETAPD HP THETAPD L THRUST ISP

(Column headings)

From Boxes 18, 19, 20, 21, 22, 23, 24, 25

Box 45

$$t_{new} = t_{old} + \Delta t$$

Box 5

(Time dependent quantities)

$$R = h + 20908000$$

Note: $g_0 = 32.17$

$$P_a = 14.7\, e^{-h/22800}$$

$$R_e = 2.0908 \times 10^7$$

$$B = \begin{cases} 1 - \dfrac{P_a}{P_e} & \text{if } \dfrac{P_a}{P_e} \leq 3 \\ -2 & \text{if } \dfrac{P_a}{P_e} > 3 \end{cases}$$

$$V_p = \frac{B}{A} V_p^0$$

$$t_B = t - \text{(length of time thrust is off)}$$

$$\frac{Tg_0}{W} = \frac{\dot{W}_F^T \left(V_e + V_p \right)(1 - \bar{\beta})}{W_0 - \dot{W}_F^T t_B}$$

$$\rho_a = (AT)\, e^{-h/22800}$$

$$c = \begin{cases} 1120 - .00414\, h & \text{if } h < 35000 \\ 975 & \text{if } h \geq 35000 \end{cases}$$

$$M = \frac{V}{c}$$

$$C_D = \begin{cases} .37 & \text{if } M \leq .9 \\ .6 + 2.3(M - 1) & \text{if } .9 < M \leq 1.1 \\ .3 + \dfrac{.6}{M} & \text{if } 1.1 < M \end{cases}$$

$$\frac{Dg_0}{W} = \frac{E C_D\, \rho_a\, V^2}{W_0 - \dot{W}_F^T t_B} \qquad \text{(E found in Box 3)}$$

$$g = \frac{g_0 R_e^2}{R^2}$$

integrate from t to t + Δt:

$$\dot{V} = \frac{Tg_0}{W} - \frac{Dg_0}{W} - g \cos \beta$$

$$\dot{\beta} = \frac{g}{V} \sin \beta - \frac{V}{R} \sin \beta$$

$$\dot{h} = V \cos \beta$$

$$\dot{\theta} = \frac{V}{R} \sin \beta$$

To Box 6

UNCLASSIFIED

From Box 5

Box 6

$$L = (h + 20908000)^2 \, \dot{\theta}$$

$$W_F = W_F^T - \dot{W}_F^T t_B$$

$$P_a = 14.7 \, e^{-h/22800}$$

$$B = \begin{cases} 1 - \dfrac{P_a}{P_e} & \text{if } \dfrac{P_a}{P_e} \le 3 \\[2mm] -2 & \text{if } \dfrac{P_a}{P_e} > 3 \end{cases}$$

$$V_p = \frac{B}{A} \, V_\rho^0$$

$$I_{sp} = \frac{V_e + V_p}{32.17}$$

$$T = I_{sp} \, \dot{W}_F^T \, (1 - \bar{\beta})$$

Box 7

Print (in appropriate column)

t   V   β   h   θ   V̇   β̇   ḣ   θ̇   L   T   I_sp

Box 28

Has missile hit ground?   No ←   Yes → To Box 1

Box 18

Branch 1   Branch 2   Exit 2   Branch 3   Exit 2   Branch 4   Exit 1

Exit 2   Exit 1   Exit 1   Exit 1   Exit 3   Exit 2

No   No   Yes

Box 22   **Box 23**   **Box 24**

Has missile run out of fuel?

Has kickover time been reached?

Has cutoff height been reached?   No

Has height h_2 been reached?

Has cut-on height been reached?

Is $\dot{h} \le 0$?

Yes   Yes   Yes   No   Yes   No   Yes   Yes   No

Box 19

Turn off thrust.
Record time at
    which thrust
    is turned off.
Set Branch 1 to
    Exit 2.

Box 20

$\beta = \beta_k$
Set Branch 2
to Exit 2

Box 21

Turn off thrust.
Record time at
    which thrust
    is turned off.
Set Branch 3 to
    Exit 2.

Box 25

Turn on thrust.
Record time at
    which thrust
    is turned on.
Set Branch 4 to
    Exit 3.

To Box 45   To Box 8

Box 22

Box 8

Note: $R_e = 2.0908 \times 10^7$

$t_2 = t$

$L_0 = \left(h + R_e\right) V \sin \beta$

$U_0 = V \cos \beta$ } Values at $t = 0$

$h_0 = h$

$L_3 = R_e \sqrt{32.17 \left(h_f + R_e\right)}$

$\lambda_1^0 = \dfrac{W_0 - \dot{W}_F^T t_B}{2.6\, W\left(V_e + V_p\right)}$

$\lambda_2^0 = -\dfrac{1}{2} \lambda_1^0$

$\lambda_3^0 = \dfrac{.71}{V_e + V_p}$

From Boxes 26, 27

Box 9

$\lambda_1 = \lambda_1^0$

$\lambda_2 = \lambda_2^0$ } Values at $t = t'$

$\lambda_3 = \lambda_3^0$

$\Delta\lambda_1 = \begin{cases} .175\, |\lambda_1| & \text{if } \lambda_1 \neq 0 \\ .0001 & \text{if } \lambda_1 = 0 \end{cases}$

$\Delta\lambda_2 = \begin{cases} .175\, |\lambda_2| & \text{if } \lambda_2 \neq 0 \\ .0001 & \text{if } \lambda_2 = 0 \end{cases}$

$\Delta\lambda_3 = \begin{cases} .175\, |\lambda_3| & \text{if } \lambda_3 \neq 0 \\ .0001 & \text{if } \lambda_3 = 0 \end{cases}$

Set Branch 5 to Exit 1.

— From Boxes 14, 15, 16

Box 10

$L = L_3$

$U = 0$ } Values at $t = t'$

$h = h_f$

$t' = \dfrac{W_0 - \dot{W}_F^T t_B}{W} \left(V_e + V_p\right) \sqrt{\lambda_1^2 + \lambda_2^2}$

Box 11

Is $t' > 0$?   Yes   To Box 12

No

Select a new set of λ's:

$\lambda_1^0 = .005$	$\lambda_2^0 = .0025$	$\lambda_3^0 = -.00004$
$\lambda_1^0 = .004$	$\lambda_2^0 = .0025$	$\lambda_3^0 = -.00004$
$\lambda_1^0 = .003$	$\lambda_2^0 = .0025$	$\lambda_3^0 = -.00004$
$\lambda_1^0 = .002$	$\lambda_2^0 = .0025$	$\lambda_3^0 = -.00004$
$\lambda_1^0 = .001$	$\lambda_2^0 = .0025$	$\lambda_3^0 = -.00004$

Box 26

Have all sets been tried?

No   Yes

To Box 9

Box 27

Print:

T is negative.

To Box 1

UNCLASSIFIED

From Box 11

Box 12

$R = h + R_e$

Note: $g_0 = 32.17$

$R_e = 2.0908 \times 10^7$

$$\eta = \frac{g_0 T}{\left[W_0 - \dot{W}_F^T\left(t + t_B\right) \right] \sqrt{\lambda_1^2 + \lambda_2^2}}$$

Integrate from $t = t'$ to $t = 0$:

$$\dot{L} = R\lambda_1 \eta$$

$$\dot{U} = \frac{L^2}{R^3} - \frac{g_0 R_e^2}{R^2} + \lambda_2 \eta$$

$$\dot{h} = U$$

$$\dot{\lambda}_1 = \frac{U\lambda_1}{R} - \frac{2L\lambda_2}{R^2}$$

$$\dot{\lambda}_2 = -\lambda_3$$

$$\dot{\lambda}_3 = \frac{\lambda_2}{R^3}\left(\frac{3L^2}{R} - 2g_0 R_e^2 \right) - \frac{\lambda_1^2 \eta}{R}$$

Box 13

$R = h + R_e$

Note: $g_0 = 32.17$

$R_e = 2.0908 \times 10^7$

$$\eta = \frac{g_0 T}{\left[W_0 - \dot{W}_F^T\left(t + t_B\right) \right] \sqrt{\lambda_1^2 + \lambda_2^2}}$$

Integrate from $t = 0$ to $t = t'$:

$$\dot{L} = R\lambda_1 \eta$$

$$\dot{U} = \frac{L^2}{R^3} - \frac{g_0 R_e^2}{R^2} + \lambda_2 \eta$$

$$\dot{h} = U$$

$$\dot{\lambda}_1 = \frac{U\lambda_1}{R} - \frac{2L\lambda_2}{R^2}$$

$$\dot{\lambda}_2 = -\lambda_3$$

$$\dot{\lambda}_3 = \frac{\lambda_2}{R^3}\left(\frac{3L^2}{R} - 2g_0 R_e^2 \right) - \frac{\lambda_1^2 \eta}{R}$$
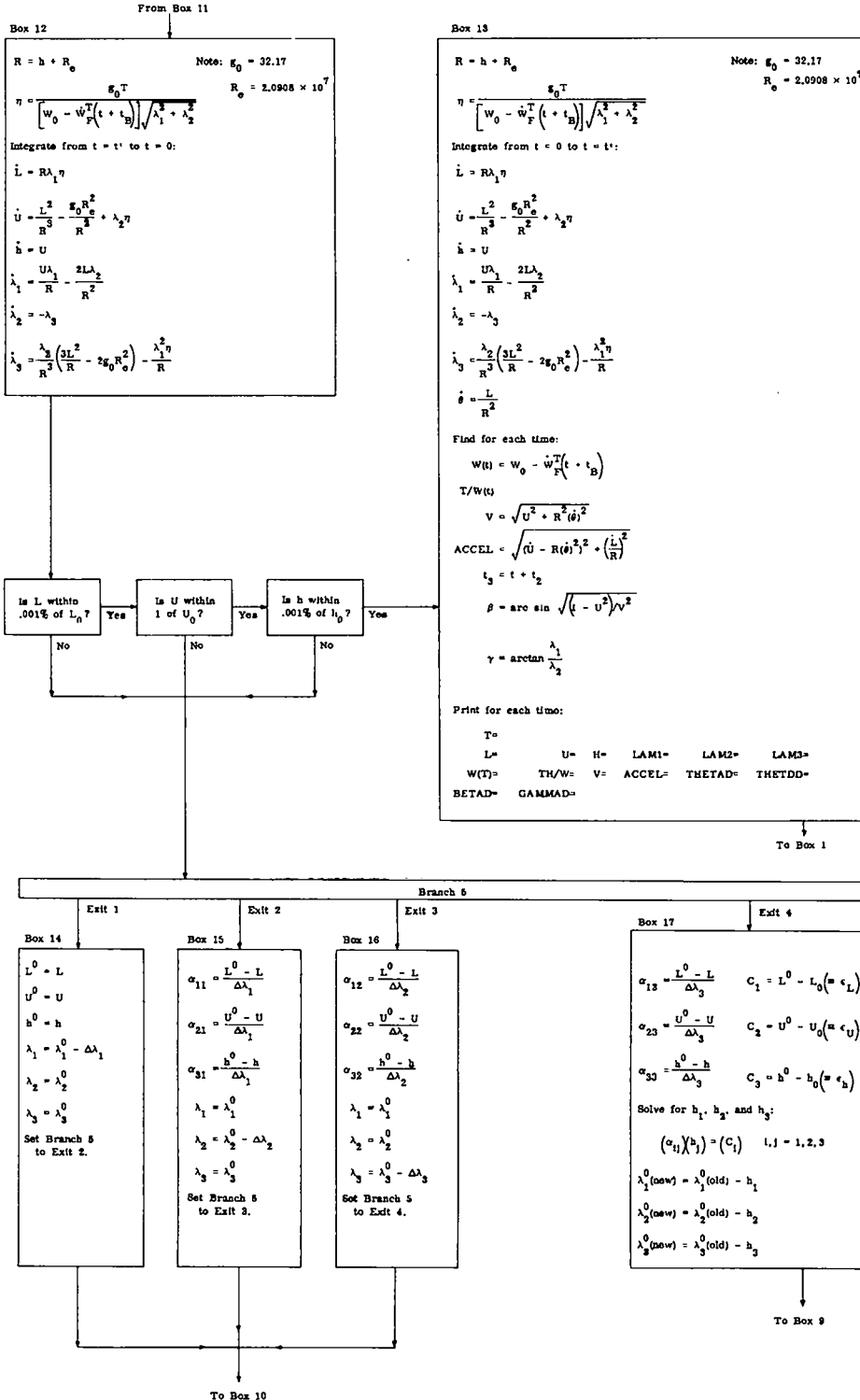
$$\dot{\theta} = \frac{L}{R^2}$$

Find for each time:

$$W(t) = W_0 - \dot{W}_F^T\left(t + t_B\right)$$

$T/W(t)$

$$V = \sqrt{U^2 + R^2(\dot{\theta})^2}$$

$$ACCEL = \sqrt{\left(\dot{U} - R(\dot{\theta})^2\right)^2 + \left(\frac{\dot{L}}{R}\right)^2}$$

$$t_3 = t + t_2$$

$$\beta = \arcsin \sqrt{\left(t - U^2\right)/V^2}$$

$$\gamma = \arctan \frac{\lambda_1}{\lambda_2}$$

Print for each time:

T=

L=   U=   H=   LAM1=   LAM2=   LAM3=

W(T)=   TH/W=   V=   ACCEL=   THETAD=   THETDD=

BETAD=   GAMMAD=

To Box 1

Is L within .001% of L_0?   Yes   Is U within 1 of U_0?   Yes   Is h within .001% of h_0?   Yes

No   No   No

Branch 5

Exit 1   Exit 2   Exit 3   Exit 4

Box 14

$L^0 = L$

$U^0 = U$

$h^0 = h$

$\lambda_1 = \lambda_1^0 - \Delta\lambda_1$

$\lambda_2 = \lambda_2^0$

$\lambda_3 = \lambda_3^0$

Set Branch 5 to Exit 2.

Box 15

$$\alpha_{11} = \frac{L^0 - L}{\Delta\lambda_1}$$

$$\alpha_{21} = \frac{U^0 - U}{\Delta\lambda_1}$$

$$\alpha_{31} = \frac{h^0 - h}{\Delta\lambda_1}$$

$\lambda_1 = \lambda_1^0$

$\lambda_2 = \lambda_2^0 - \Delta\lambda_2$

$\lambda_3 = \lambda_3^0$

Set Branch 5 to Exit 3.

Box 16

$$\alpha_{12} = \frac{L^0 - L}{\Delta\lambda_2}$$

$$\alpha_{22} = \frac{U^0 - U}{\Delta\lambda_2}$$

$$\alpha_{32} = \frac{h^0 - h}{\Delta\lambda_2}$$

$\lambda_1 = \lambda_1^0$

$\lambda_2 = \lambda_2^0$

$\lambda_3 = \lambda_3^0 - \Delta\lambda_3$

Set Branch 5 to Exit 4.

Box 17

$$\alpha_{13} = \frac{L^0 - L}{\Delta\lambda_3} \qquad C_1 = L^0 - L_0\left(= \epsilon_L\right)$$

$$\alpha_{23} = \frac{U^0 - U}{\Delta\lambda_3} \qquad C_2 = U^0 - U_0\left(= \epsilon_U\right)$$

$$\alpha_{33} = \frac{h^0 - h}{\Delta\lambda_3} \qquad C_3 = h^0 - h_0\left(= \epsilon_h\right)$$

Solve for h_1, h_2, and h_3:

$$\left(\alpha_{ij}\right)\left(h_j\right) = \left(C_i\right) \qquad i, j = 1, 2, 3$$

$\lambda_1^0(\text{new}) = \lambda_1^0(\text{old}) - h_1$

$\lambda_2^0(\text{new}) = \lambda_2^0(\text{old}) - h_2$

$\lambda_3^0(\text{new}) = \lambda_3^0(\text{old}) - h_3$

To Box 9

To Box 10